Name: ____ Date: __________________

Graphs of Radical Functions

Objective

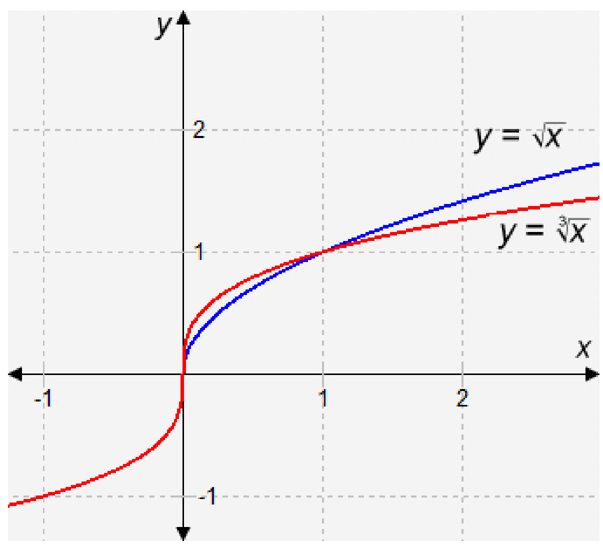
In this lesson, you will analyze key features of radical functions algebraically and graphically.

Analyzing Graphs of Radical Functions

The most common __________ functions are the square root and the cube root functions.

The __________ functions for these two function types are shown on the graph.

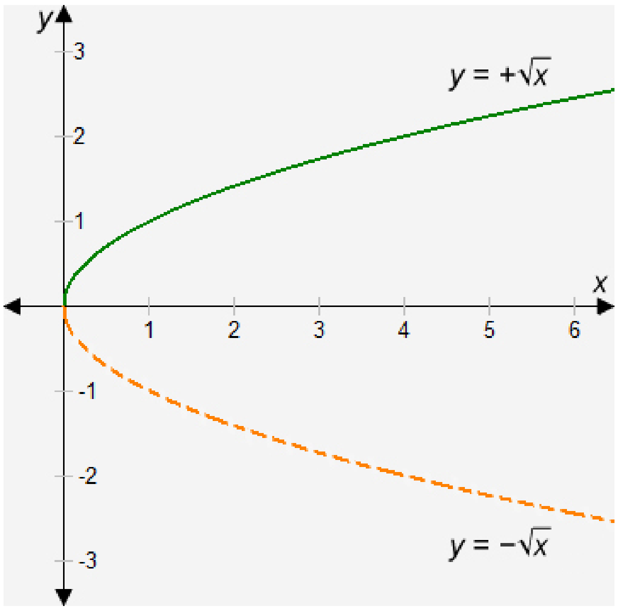
- All radical functions are ______________, (strictly increasing or decreasing). Both parent functions are strictly ______________.
- The end behavior of radical functions always goes to positive or negative __________. For the parent functions, as the value of x increases, the value of y ___________ without bound.



SQUARE ROOT FUNCTIONS

- always have an endpoint, called the _________, which can be considered a minimum or a ______________ of the function. The vertex of the parent function is at (___,___).
- There may be one or no x- and y-intercepts depending on the position of the graph. The parent function has one x-intercept (also called a ______) and one y-intercept, both located at (___,___).

Square root functions have range and domain restrictions which depend depending on their position on the coordinate plane. Consider the relation $y^2 = x$ solved for y to get $y = \pm\sqrt{x}$.

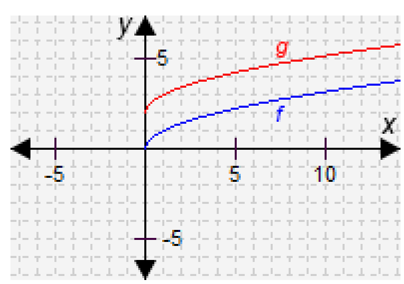


When both the positive and negative square roots are considered, the graph does not pass the ____________ line test to be a function. Therefore, we restrict the __________ of $y = \pm\sqrt{x}$ to only ______________ y-values, which is called the principal square root.

The ____________ of a square root function is limited to values that result in ________ numbers. (The values under the radical sign must be ≥ 0.) The square root function is continuous everywhere on its _______________.

SQUARE ROOT TRANSFORMATIONS

The standard form of a transformed square root function is $y = a\sqrt{b(x-h)} + k$.

<table>
<tr><td>Vertical Translation
• ____________ a constant, k, to a function shifts it up k units; subtracting k shifts it ___________.
• Shape stays the same, but the ___________ changes.</td><td>
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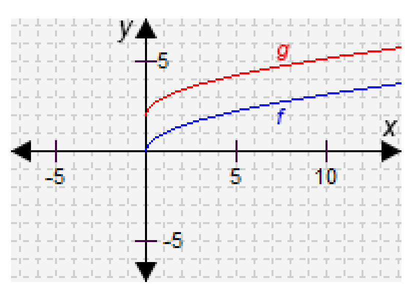
<tr><td>Horizontal Translation
• Adding a constant, h to the __________ shifts it left h units; subtracting shifts it __________.
• Shape stays the same, but the _____________ changes.</td><td>
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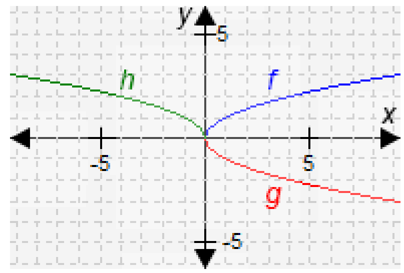
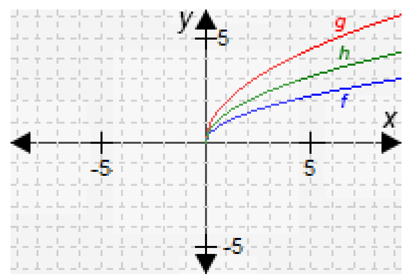
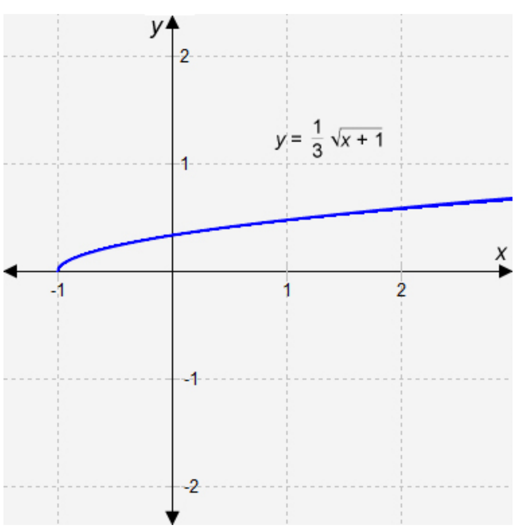
Reflection • Multiplying the ____________ by a negative constant, reflects across the *x*-axis; multiplying the ___________ by a negative constant reflects the function across the *y*-axis	
Dilation • Multiplying the ____________ by $\|a\| > 1$ vertically stretches it _________ from the ___-axis. If $0 < \|a\| < 1$, it vertically compresses ___________ the ___-axis • Multiplying the _______ by $\|b\| > 1$ horizontally compresses the function ___________ the ___-axis. If $0 < \|b\| < 1$, it horizontally stretches _________ from the ___-axis.	

Example: Consider the graph of $y = \frac{1}{3}\sqrt{x+1}$.

The graph shows a square root function that's been translated 1 unit to the _______ and ______________ compressed by a factor of $\frac{1}{3}$. Look at the key features that changed as a result.



- minimum: shifted to the _______
  from (0,0) to (____,____)
- ___________: shifted to the left
  from [0, ∞) to [____, ∞)
- ___-intercept: shifted to the left
  from x = ___ to x = ____
- ___-intercept: since the graph shifted _______,
  now located at $y = \frac{1}{3}$

The range and end behavior have not changed and are the same as for the parent function.

CUBE ROOT FUNCTIONS

Key features of the parent cube root function:

- The function always has ______ *x*-intercept and ______ *y*-intercept. In the parent function, the *x*- and *y*-intercepts happen to be at the same point, (____,____).
- The domain and range for any cube root function are all ________ numbers.
- Every cube root function is _______________.
- For the parent function: as *x* approaches positive infinity, *y* approaches ____________ infinity, and as *x* approaches negative infinity, *y* approaches ______________ infinity.
- Unlike square root functions, cube root functions have _______________ symmetry. The point of rotational symmetry is called the _________ point which centers the function, like a vertex. On the parent function, the pivot point is the __________.

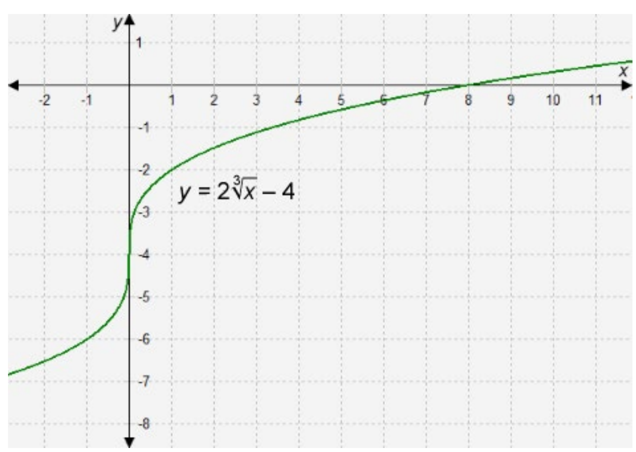
The standard form of a transformed cube root function is $y = a\sqrt[3]{b(x-h)} + k$. The values of *a*, *b*, *h*, and *k* transform cube functions the same way they transformed square root functions.

Example: Consider the graph of $y = 2\sqrt[3]{x} - 4$.

The graph shows a cube root function translated ________ 4 units and vertically ______________ by a factor of 2. Look at the key features that changed as a result.

- pivot point: shifted from (0, 0) to (____,____)
- ___-intercept: shifted from *x* = 0 to *x* = ____
- ___-intercept: shifted from *y* = 0 to *y* = _____

All other key features of this function are the same as for the parent cube root function.



Analyzing Tables of Radical Functions

The table gives values for the square root function $f(x) = 2\sqrt{x+3}$.

x	$f(x) = 2\sqrt{x+3}$
-3	$f(-3) = 2\sqrt{(-3)+3}$ = 0
-2	$f(-2) = 2\sqrt{(-2)+3}$ = 2
-1	$f(-1) = 2\sqrt{(-1)+3}$ ≈ 2.8284
0	$f(0) = 2\sqrt{(0)+3}$ ≈ 3.4641
1	$f(1) = 2\sqrt{(1)+3}$ = 4
2	$f(2) = 2\sqrt{(2)+3}$ ≈ 4.4721
3	$f(3) = 2\sqrt{(3)+3}$ ≈ 4.8990

From the table, the *x*-intercept occurs at (____, 0) and the *y*-intercept is approximately (0, ____________).

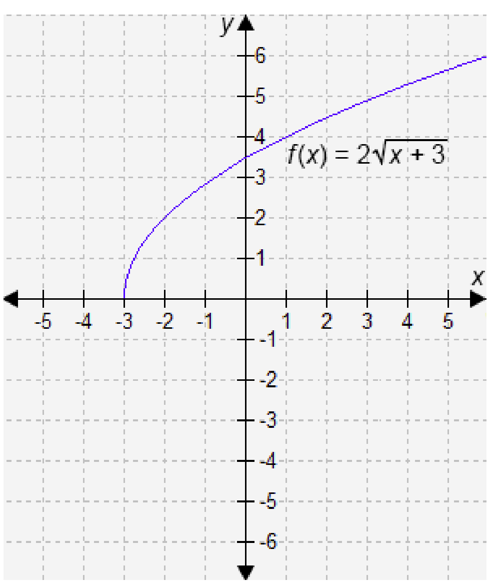
Because the output values are continually getting ___________, the function is ______________.

The end behavior will go to positive infinity.

The starting point of the function is also the ______________, which occurs at (____, ____).

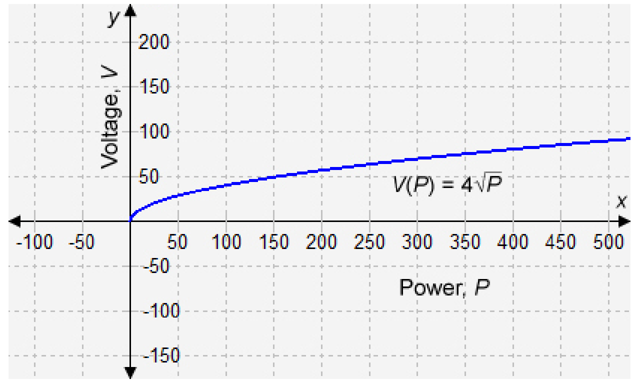
KEY FEATURES OF A SQUARE ROOT FUNCTION

From the equation, the parent square root function has been translated to the left 3 units and vertically stretched by a factor of 2 to get $f(x) = 2\sqrt{x+3}$. Look at the key features.



- The ___________ or endpoint, of function *f* is (-3,0). Therefore, the domain is [____, ∞).
- The range is [____, ∞).
- The ___-intercept of the parent function has been translated 3 units to the left to (_____,____).
- Since $f(0) = 2\sqrt{0+3} = 2\sqrt{3} \approx 3.4641$, the ___-intercept is approximately (0, __________).
- The function is continuous over its domain and strictly _________________. As *x* approaches positive infinity, *f*(*x*) approaches ______________ infinity.

Example: Suppose the voltage of a speaker can be modeled by the equation $V(P) = 4\sqrt{P}$, where *P* is the power of the speaker.



→ (0, 0) is the ____________ of the function. This means that if there is no power, the audio system will have no ____________.

→ The *x*-values can't be less than 0 because there can't be __________ power. Also, it isn't viable to say the *x*-values go on forever. They will need to be cut off at the highest level of _________ the system can handle.

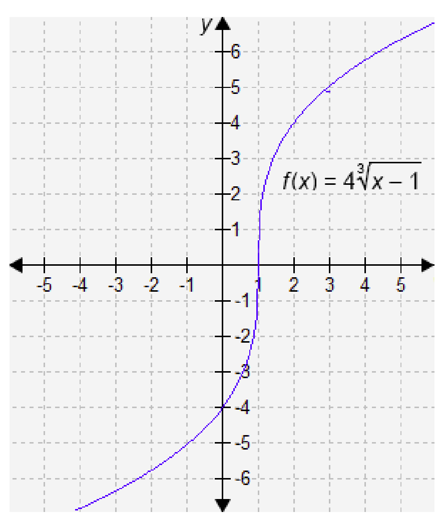
→ Likewise, the range can't contain ____________ numbers and there is a maximum voltage for a particular audio system. So, the range cannot contain all positive numbers.

Example: The table shows the input values and output values for the function $f(x) = 4\sqrt[3]{x-1}$.

x	$f(x) = 4\sqrt[3]{x-1}$
-3	$f(-3) = 4\sqrt[3]{(-3)-1}$ ≈ -6.3496
-2	$f(-2) = 4\sqrt[3]{(-2)-1}$ ≈ -5.7690
-1	$f(-1) = 4\sqrt[3]{(-1)-1}$ ≈ -5.0397
0	$f(0) = 4\sqrt[3]{(0)-1}$ = -4
1	$f(1) = 4\sqrt[3]{(1)-1}$ = 0
2	$f(2) = 4\sqrt[3]{(2)-1}$ = 4
3	$f(3) = 4\sqrt[3]{(3)-1}$ ≈ 5.0397

Key Features:

- *x*-intercept: (____,____)
- *y*-intercept: (____,____)
- pivot point: (____,____)
- The function is _____________ without bound.



- domain and range: all _______ numbers
- end behavior: as *x* approaches __________ infinity, *f*(*x*) approaches positive infinity.
  As *x* approaches negative infinity, *f*(*x*) approaches _____________ infinity.