Name: ______________________________ Date: ______________

Triangle Properties

Objective

In this lesson, you will determine different triangle properties and use them to solve problems involving triangles.

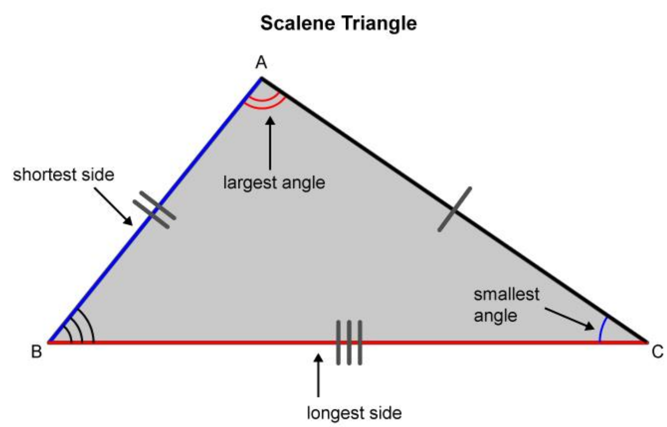
The Side-Angle Relationship in Triangles

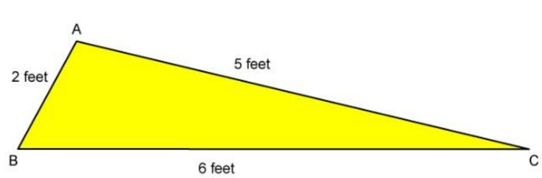
A triangle is formed by two types of geometric figures:

- **Line** __ **segments** ____ form the sides of the triangle; their **lengths** ____ determine the triangle's size.
- Angle measures define the **shape** ___ of the triangle.

Each side of a **scalene** ___ triangle has a **different** ___ length, and each **angle** ___ has a different measure.

- The side opposite the **largest** ____ angle is the **longest** ___ side.
- The side **opposite** ____ the smallest **angle** ____ is the shortest side.

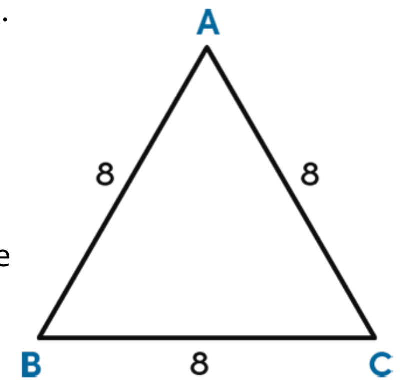




- ➢ Here, side **AB** __ has the shortest length, so ∠**C** __ has the smallest measure.
- ➢ Side **AC** ___ has the shortest length of the two remaining sides, so ∠**B** ___ has the next smallest measure.
- ➢ The angles in order from least to greatest are ∠**C** __, ∠**B** __, and ∠**A** __.

An equilateral triangle is a triangle with **three** congruent sides.

- the size of the angle **opposite** a side determines the **length** of a side
- **all** angles are also **congruent**.
- the three congruent angles in an equilateral triangle each have a measure of **60**°.

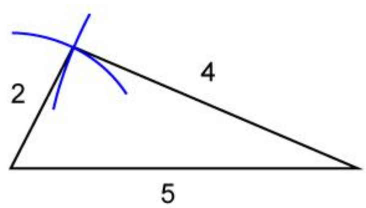
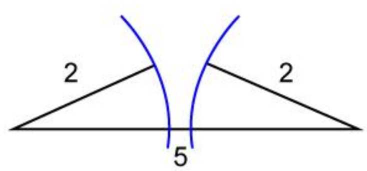
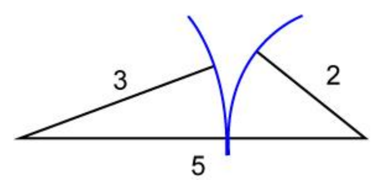


An isosceles triangle has **two** congruent sides.

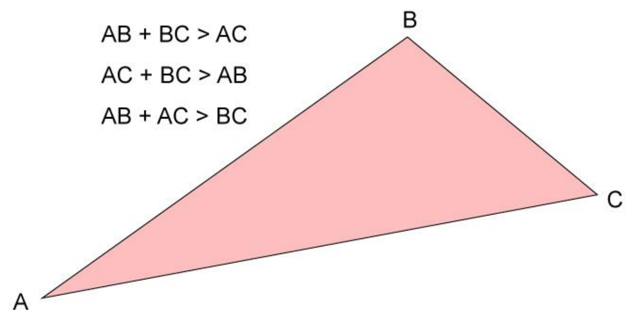
- the lengths of these sides are **equal**
- the angles **opposite** these sides are also equal
- an isosceles triangle has **exactly** **two** congruent angles opposite the **congruent** sides.

The Triangle Inequality Theorem

Not all groups of **three** segment lengths can form a **triangle**.

Case 1  $2 + 4 > 5$	Case 2  $2 + 2 < 5$	Case 3  $3 + 2 = 5$
The two shorter sides meet **opposite** the longest side to form a triangle. The **sum** of the two shorter side lengths is **greater** than the longest side length.	The sum is **less** than the **longest** side length. The two **shorter** sides would **never** meet to form a triangle.	The sum is **equal** to the **longest** side length. The two **shorter** sides would **not** meet until they coincide with the third side.

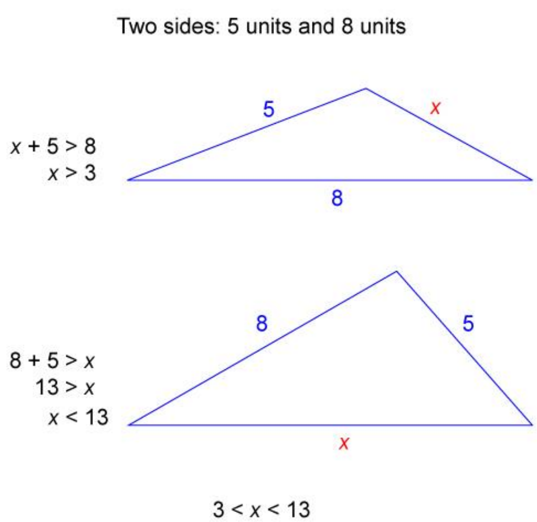
The triangle inequality theorem states that the **sum** of the lengths of any two sides of a triangle is **greater** than the length of the third side.



Possible Side Lengths in Triangles

The **triangle** **inequality** theorem allows us to use **two** side lengths to determine the range of possible values for the **third** side length of a triangle.

Consider a triangle with one side of length 5 units and another of length 8 units. Determine the range in which the length of the third side must lie.



- Draw a diagram to represent the **minimum** value of the third side, x. Show the **smallest** side as x.
- Use the triangle **inequality** theorem to write an inequality for the minimum value of the third side and solve for **x**.
- Sketch a second diagram to represent the **maximum** value of the third side. Show the **largest** side as x.
- Write an **inequality** for the maximum value of the third side and **solve** for x.
- Use the minimum and maximum values to write a **compound** inequality for **x**, representing the **range** for the length of x.