Name: ______________________________ Date: ______________

Graphing Polynomial Functions

Objective

In this lesson, you will analyze key features of polynomial functions algebraically and graphically.

Key Features of Polynomial Functions

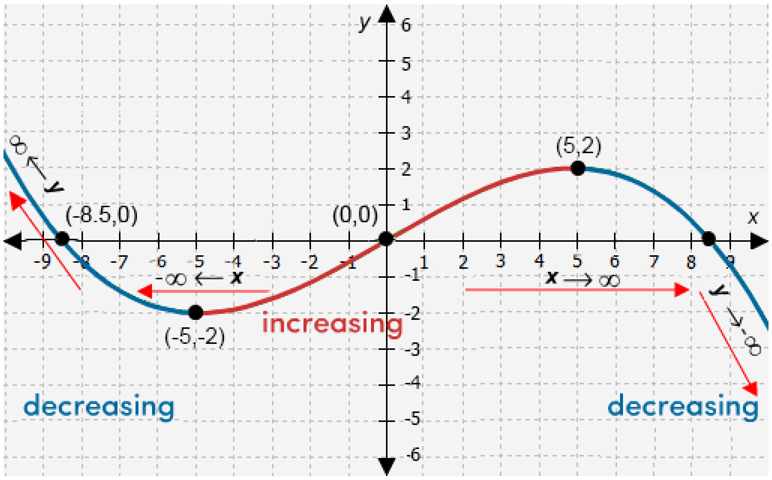
Key Features:

- extreme values: **relative** and absolute maximum and **minimum** points
- zeros - points where the function crosses the ***x***-axis (also known as the *x*-intercept)
- *y*-intercept - point where the function crosses the ***y***-axis
- domain - all the possible ***x*** values for the function
- range - all the possible ***y*** values for the function
- intervals where the graph is increasing and where it is decreasing
- end behavior - a description of the values of the graph as the independent variable approaches infinity or negative infinity

		Sign of the Lead Coefficient	
		Positive	**Negative**
Polynomial Degree	**Odd**	values approach $-\infty$ on the left and ∞ on the right	values approach ∞ on the left and $-\infty$ on the right
	Even	values approach ∞ on both the left and the right	values approach $-\infty$ on both the left and the right

Example:
The figure shows the graph of a cubic polynomial.

Let's identify the key features.



By looking at the graph, we can approximate that

- ⇒ the relative minimum is (**-5**, **-2**) and the relative maximum is (**5**, **2**), and
- ⇒ there are no absolute extrema because the graph extends toward infinity and negative infinity in the *y* directions.

The graph decreases and increases between these **extrema**.

- ⇒ It is decreasing on the intervals (−∞, **-5**) and (**5**, ∞).
- ⇒ It is increasing on the interval (**-5**, **5**).

Since the function switches direction at these points, their *x*-values are the endpoints of the intervals but are not included.

The zeros mark where the value of the function changes signs.

- ⇒ The zeros are (-8.5,0), (0,0) and (**8.5**, **0**).
- ⇒ The function is positive on the intervals (−∞, **-8.5**) and (**0**, **8.5**).
- ⇒ The graph is negative on the intervals (**-8.5**, **0**) and (**8.5**, ∞).

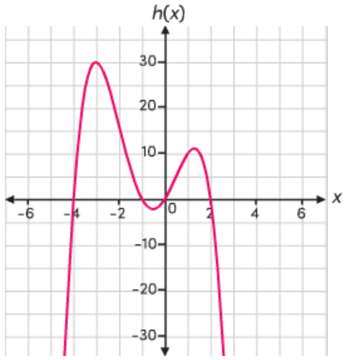
Find the end behavior.

- ⇒ As the *x*-values approach infinity, the function approaches **negative** infinity.
- ⇒ As the *x*-values approach negative infinity, the function approaches infinity.

To determine the domain and the range, look at the values that are possible for the *x*-coordinates and the *y*-coordinates.

- ⇒ This is a **continuous** function.
- ⇒ Both the domain and the range are all **real** numbers.

Question

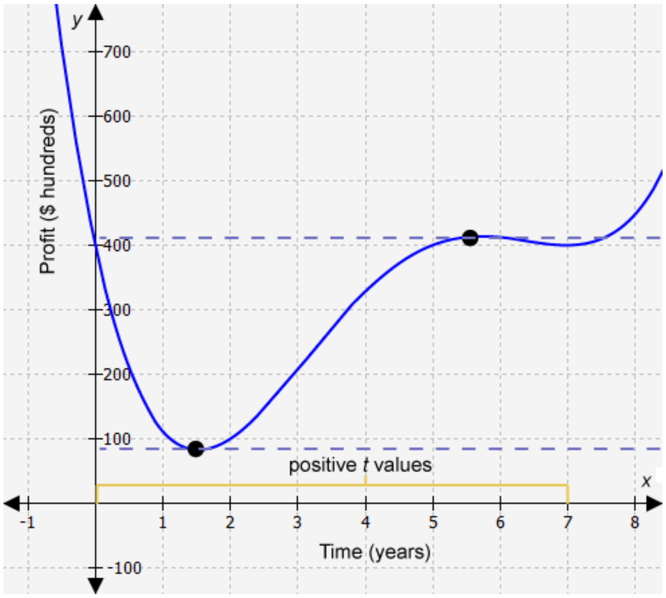


We can use this graph to determine the key features of polynomial function *h*.

This function has an absolute **maximum** value.

So, the range of the function is from **negative** infinity to **30**.

This polynomial models a real-world situation.



Golden Incorporated went through its financial records for the last seven years and modeled its profits with a quartic function.

The graph shows the profit, $p(t)$, in hundreds of dollars as a function of time, t, in years.

If we consider the graph out of context, both the domain and the range are all real numbers.

However, when we consider the graph in the context of time and profit, there are some restrictions.

- The domain is restricted to positive numbers because time can never be **negative**.
- The **range** will be bounded between the lowest and highest profits the company had in the 7 years being modeled. (The graph's end behavior indicates values outside these bounds would not be valid or realistic predictions.)

  So, because of the context, the domain and range are restricted.

  Domain: [**0**, **7**]   Range: [80,420]

We can use interval notation, set builder notation, or inequalities to express the domain, range, or intervals where the function has certain behavior.

Ex. If the range of function is from negative infinity to 4, we can use any of these:

$(-\infty, 4]$ or $-\infty < y \leq 4$ or $\{y \mid -\infty < y \leq 4\}$.

Now, let's look at a polynomial function given as a table of values.

x	$f(x)$
0	0
1	5.5
2	2.1
3	0
4	-0.7
5	-1.2
6	3.8

Identify where the function is increasing or decreasing.

⇒ This function increases on the intervals (0,**1**) and (**5**,6).

⇒ The function is decreasing on the interval (**1**,**5**).

Finding zeros from a table can be trickier. In addition to the zeros given directly, we look for where the function changes between positive and negative values.

⇒ This table reveals two of the zeros (0,0) and (**3**,0).

⇒ Because all polynomials are **continuous**, we know the function must cross the ***x***-axis at some point between 5 and **6**. Since we don't have any more data, we can only estimate that the third zero is somewhere between $x = 5$ and $x =$ **6**.

To determine the domain and the range, we can include only the values in the table. We cannot assume behavior beyond the table.

⇒ The domain is [**0**, **6**].

⇒ The lowest value in the table is **-1.2**, and the highest value is **5.5**.

⇒ So, the range of the function is $-1.2 \leq f(x) \leq 5.5$.

To find the extrema from a table, we must locate all of the relative maximums and minimums.

⇒ (0,0), (1,5.5), (5,-1.2) and (6,3.8)

Since the table doesn't provide a full picture of the function, these points are our best **estimates** for the relative extrema.

We can use these points to approximate and **interpret** features of a function when only a table is given.

A polynomial function will naturally have peaks and valleys. So, there may be more extrema than the table shows.

For example, we don't know what the function looks like before $x = 0$.

There also could be values close to $x = 1$ that are greater than 5.5 and values close to $x = 5$ that are less than -1.2.

Graphs of Polynomial Functions

We'll use key features to create graphs from the symbolic form of a function.

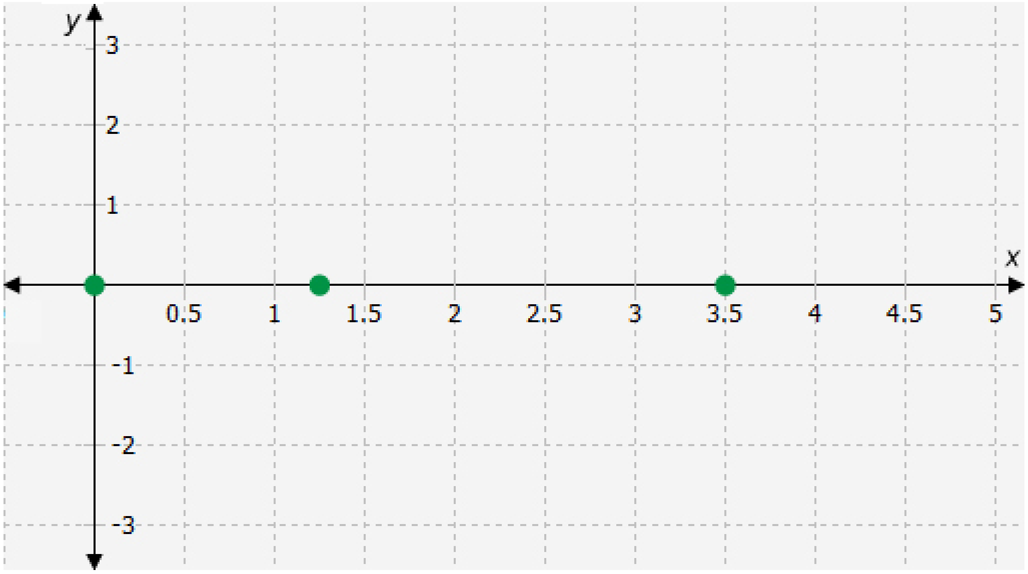
1. Find the zeros of the function and plot them on a graph.

Ex. $p(x) = 18x^3 - 63x^2 - 12x(2x - 7) = 3x(2x - \underline{\ 7\ })(3x - \underline{\ 4\ })$

By setting each factor equal to zero and solving for x,

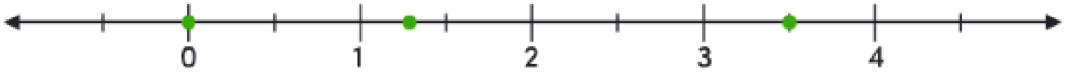
we can determine the zeros of $p(x)$: are $x = \underline{\ 0\ }$, $x = \frac{4}{3}$, and $x = \frac{7}{2}$.

Here we used factoring, but we can also use synthetic division to find zeros.



2. Sketch a rough graph of a function by testing values between the zeros to find the intervals where the function is positive or negative.

Ex. $p(x) = 3x(2x - 7)(3x - 4)$



interval 1 $(-\infty,\ 0)$	interval 2 $\left(0,\ \frac{4}{3}\right)$	interval 3 $\left(\frac{4}{3},\ \frac{7}{2}\right)$	interval 4 $\left(\frac{7}{2},\ \infty\right)$
Since $p(-1) = \underline{-189}$, the function is negative.	Since $p(1) = \underline{15}$, the function is positive.	Since $p(2) = \underline{-36}$, the function is negative.	Since $p(4) = \underline{96}$, the function is positive.

3. Now, find the **y**-intercept by finding $p(0)$.

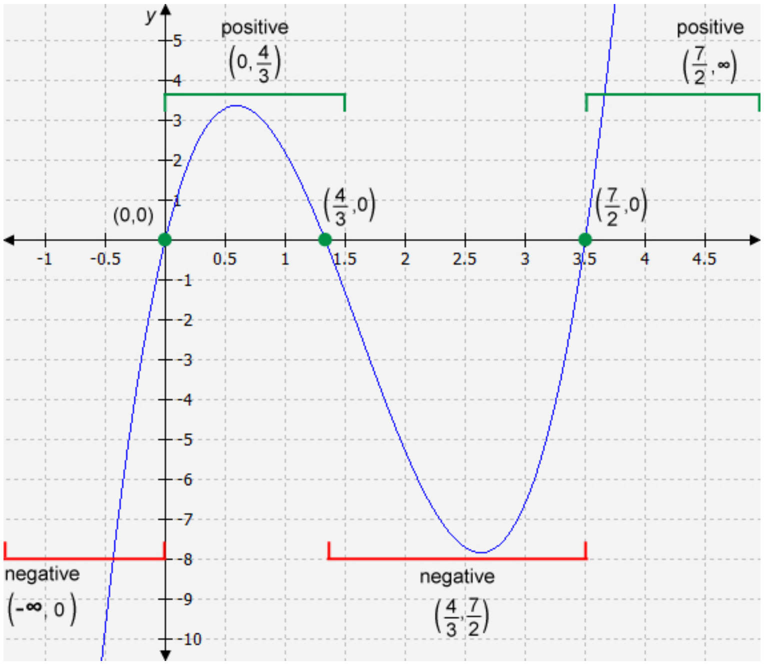
Ex. $p(x) = 3x(2x - 7)(3x - 4)$

$p(0) = 3(0)(2(0) - 7)(3(0) - 4)$

$= 0$

The **y**-intercept is at (**0**, **0**).

4. Use this information to make a rough sketch of the graph.



Up to this point, we've done everything by hand.

We can also use **graphing** technology to pinpoint the extrema on the graph.

Ex. Using a graphing tool, we find a relative maximum of the function $p(x)$ is at (**0.591**, **22.972**) and a relative minimum is at (**2.631**, **-53.404**).