

# Applications of Ratio and Percent



## Objective

In this lesson, you will

use proportional relationships to solve ratio and percent problems.

## Solving Ratio Problems

if two ratios,  $a:b$  and  $c:d$ , are in a proportional relationship

$$\Rightarrow \frac{a}{b} = \frac{c}{d} \Rightarrow$$

can find an unknown value in a **proportional** relationship if the other three values are known

Ryan discovers that the ratio of tigers to lions at a popular national park is 3:4. There are 24 lions in the park, but he doesn't know the number of tigers. He can find the number of tigers.

Suppose the park has  $x$  tigers. The ratio of tigers to lions is 3: $4$ .

The actual number of tigers to the actual number lions is  $x$ :24. These two ratios must be equal.

$$\frac{\text{number of tigers}}{\text{number of lions}} = \frac{3}{4} = \frac{x}{24}$$

Solve the equation to find  $x$ :

First, get rid of all the denominators.

Remove the denominator  $4$  from the fraction  $\frac{3}{4}$  by

multiplying  dividing both sides of the equation by  $\frac{4}{1}$ .

Next, remove the denominator 24 from the fraction  $\frac{4x}{24}$  by

multiplying  dividing both sides of the equation by  $\frac{24}{1}$ .

There are  $18$  tigers at the park.

$$\frac{3}{4} \times \frac{4}{1} = \frac{x}{24} \times \frac{4}{1}$$

$$\frac{3}{1} = \frac{4x}{24}$$

$$3 \times \frac{24}{1} = \frac{4x}{24} \times \frac{24}{1}$$

$$3 \times 24 = 4x$$

$$72 = 4x$$

$$x = \frac{72}{4}$$

$$x = 18$$

# Cross Multiplication

Cross multiplication is a quicker method for solving ratio and percent problems. To cross multiply two fractions, we multiply the **numerator** of each fraction by the  numerator  denominator of the other fraction.

Use cross multiplication to solve Ryan's problem and find the number of tigers in the park.

$$\frac{\text{number of tigers}}{\text{number of lions}} = \frac{3}{4} = \frac{x}{24}$$

$$\frac{3}{4} = \frac{x}{24}$$

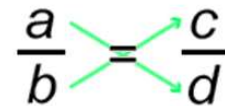
$$3 \times 24 = 4 \times x$$

$$72 = 4x$$

$$x = \frac{72}{4}$$

$$x = 18$$

Notice that 18 is the same answer we got earlier.



$$\frac{a}{b} = \frac{c}{d}$$

$$ad = bc$$



Hank is building a dog run for his dog. He wants the ratio of the length to the width of the dog run to be 5:2. If he builds the dog run so the length is 10.5 feet, which equation can be used to solve for the width,  $w$ ? What is the value of  $w$ ?

The equation that can be used to solve for the width ( $w$ ) is  $5w = 2 \times 10.5$ .

The width is  $4.2$  feet.

The ratio of length to width is 5:2. The length is 10.5 feet, and the width is  $w$ :

$$\frac{5}{2} = \frac{10.5}{w}$$

$$5w = 2 \times 10.5$$

$$w = \frac{2 \times 10.5}{5} = \frac{21}{5} = 4.2$$

The width =  $4.2$  feet.

## Solving Percent Problems

Cross multiplication can be used to solve not just fraction problems but **percent** problems too.

Peter bought a soccer ball for \$20. He paid an additional 10% sales tax. What is the total amount Peter paid for the ball?

A percentage can be expressed as a fraction with 100 as its denominator. A tax of **10**% written as a fraction is  $\frac{10}{100}$ .

That means the tax would be \$**10** on a \$100 purchase.

Let's find out how much tax Peter paid on his \$20

purchase.  $t$  is the **tax** amount.

$$\frac{\text{tax \%}}{100} = \frac{\text{tax amount}}{\text{price}}$$

$$\frac{10}{100} = \frac{t}{\boxed{20}}$$

$$100 \times t = 20 \times 10$$

$$\boxed{100t} = 200$$

$$\frac{100t}{100} = \frac{200}{100}$$

$$t = \boxed{2}$$

Use cross multiplication:

Divide both sides of the equation by 100:

The tax is \$2. Peter paid \$20 (the price) + \$2 (tax), or a total of \$**22**, for the soccer ball.

## Discounts

Discounts are expressed as a percentage of the **marked** price.

Lena wants to buy a pair of shoes. The marked price of the shoes is \$60, but the store is offering a 40% discount. Let the amount of the discount be  $d$ :

$$\frac{\text{discount \%}}{\boxed{100}} = \frac{\text{amount of discount}}{\boxed{\text{marked price}}}$$

$$\frac{40}{100} = \frac{d}{60}$$

Cross multiply:

$$60 \times 40 = d \times \boxed{100}$$

$$\boxed{2,400} = 100d$$

Divide both sides by 100:

$$\frac{2,400}{100} = \frac{100d}{100}$$

$$d = \frac{2,400}{100} = \boxed{24}$$

The amount of the discount is \$24. The discounted price is the difference of the marked price and the discount: **60** - 24 = 36. Lena should plan to pay \$**36** for the shoes.

## Percent Increase and Decrease

Quantities such as prices of items and other measures often increase or decrease. Understanding percent change (increase or decrease) gives you an idea about how much the quantities have changed.

$$\boxed{\text{percent change}} = \frac{\text{amount of change}}{\text{original amount}} \times \boxed{100}$$

To find a percent change, rearrange the quantities and the formula:



$$\frac{\text{percent change}}{100} = \frac{\text{amount of change}}{\text{original amount}}$$

The amount of change is the  difference  sum of the original amount and the new amount:



amount of decrease = original amount – new amount

amount increase = new amount – original amount

Last year, Joe earned \$10,000 working part-time at a craft store. This year, he's going to earn \$12,000. Find the percent increase in his earnings from last year to this year.

First, find the amount of increase:

$$\begin{aligned} \boxed{\text{amount of increase}} &= \text{new amount} - \text{original amount} \\ &= 12,000 - 10,000 = 2,000 \end{aligned}$$

Plug the amount of change and the original amount into the formula:

$$\frac{\text{percent change}}{100} = \frac{\text{amount of change}}{\text{original amount}}$$

$$\frac{\text{percent change}}{\boxed{100}} = \frac{2,000}{10,000}$$

Use cross multiplication:

$$\boxed{10,000} \times \text{percent change} = 2,000 \times 100$$

$$\text{percent change} = \frac{200,000}{10,000} = 20$$

Joe's salary increased 20 % between last year and this year.



Eden bought a new phone. The phone's retail price is \$199. In addition, she paid sales tax of 8% of the retail price. A few days after the purchase, she filled out a mail-in rebate form and got back \$50 from the store. How much did Eden pay for the phone after tax and the mail-in rebate?

\$268.92
  \$229.08
  \$214.92
  \$164.92

$$\frac{t}{199} = \frac{8}{100}$$

$$100t = 8 \times 199$$

$$t = \frac{8 \times 199}{100} = \frac{1,592}{100} = 15.92$$

Find the **total** price of the phone (retail price + tax):

$$\text{total price} = \$199.00 + \$15.92 = \$214.92$$

Eden got a rebate of \$50. Subtract this amount from the total price:

$$\$214.92 - \$50.00 = \$164.92$$

Eden paid \$164.92 for the phone.

## Percent Error



$$\text{percent error} = \frac{|\text{actual value} - \text{estimated value}|}{\text{actual value}} \times 100$$

A person measured a distance to be 6.25 inches, but the actual distance was 6 inches:

$$\text{percent error} = \frac{|6 - 6.25|}{6} \times 100 = 4\%$$

## Summary

How can knowing the mathematics of ratios and percents be used to help make decisions when shopping?

answers will vary