Equations of a Line

Objective

In this lesson, you will

use similar triangles to explain why the slope is the same between any two points and derive the equations y = mx and y = mx + b.

Nonproportional Lines

A straight line passing through the origin represents a **proportional** relationship. However, not all straight lines pass through the origin. Such lines represent relationships that are **not** proportional.

Lesson Activity

Find out how many cars the baseball team needs to wash before it starts making a profit. The team spent $75 setting up the car wash, and they are charging $5 per car for a wash.

A. Write an equation to represent the amount of money collected in dollars, *y*, in terms of the number of cars washed, *x*. Ignore the setup cost.

$y = 5x$

B. What do you need to do to the equation in part A to account for the setup cost?

To account for the setup cost, you need to ○ add ◉ subtract 75 from the right side of the equation obtained in part A.

C. Write an equation representing the profit made on the car wash in dollars, *y*, in terms of the number of cars washed, *x*, after expenses.

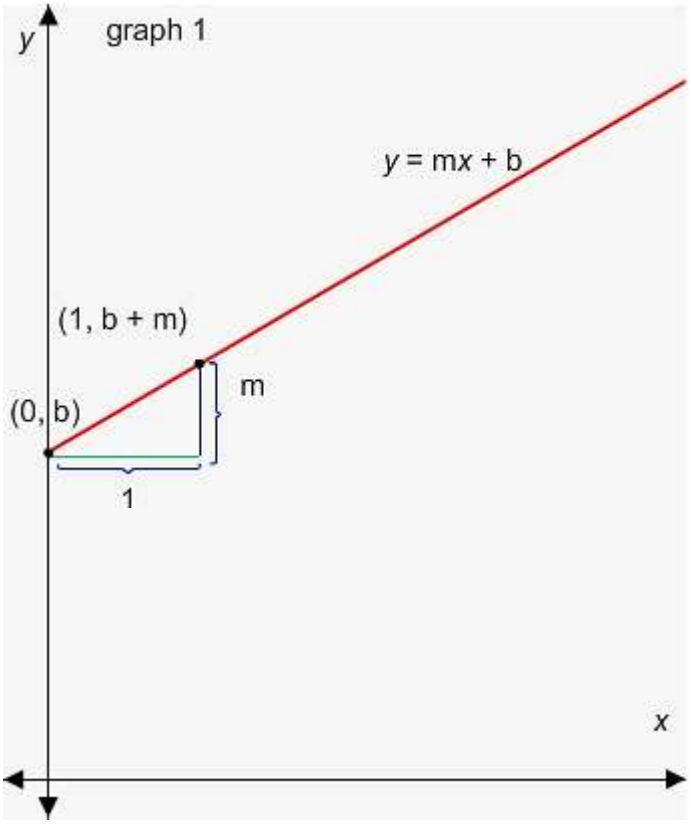
$y = 5x - 75$

D. What is the value of the profit when the baseball team washes 0 cars? What point represents this value? What does the *y*-value of this point mean in terms of the problem?

y = -75; (0,-75); If the baseball team washed no cars, it would lose $75.

E. How many cars does the baseball team have to wash to break even? What point represents this value? What does the *x*-value of this point mean in terms of the problem?

15 cars; (15,0); The baseball team would have to wash 15 cars to break even.



The general equation of a line is $y = mx + b$.

- m is the **slope** of the line
- b is the y-intercept

The line **intersects** the y-axis at the point $(0, b)$. The y-intercept **translates** the line passing through the origin **up** or **down** by b units.

The equation of a proportional relationship is $y = mx$.

The general equation for a proportional relationship is the **same** as the general equation for any line, just with b **equal** to **zero**.

Question

Lauren owes her friend $32.50 for a concert ticket. She doesn't have the cash now, but she'll earn it babysitting this weekend.

In this case, m is **8**, and b is **-32.5** because Lauren owes her friend $**32.50**.

If Lauren gets paid $8 an hour to babysit, the equation that represents y, how much money will she have after paying back her friend, in terms of x, the number of hours she spends babysitting this weekend, is:

$y =$ **8** x **– 32.5**

Positive and Negative Slope

Recall that the slope of a line is the rise over the run from one point to the other.

$$m = \frac{\text{rise}}{\text{run}}$$

Rise is the number of units we move up or down from one point to another. On a graph, rise is the change in the y-values.

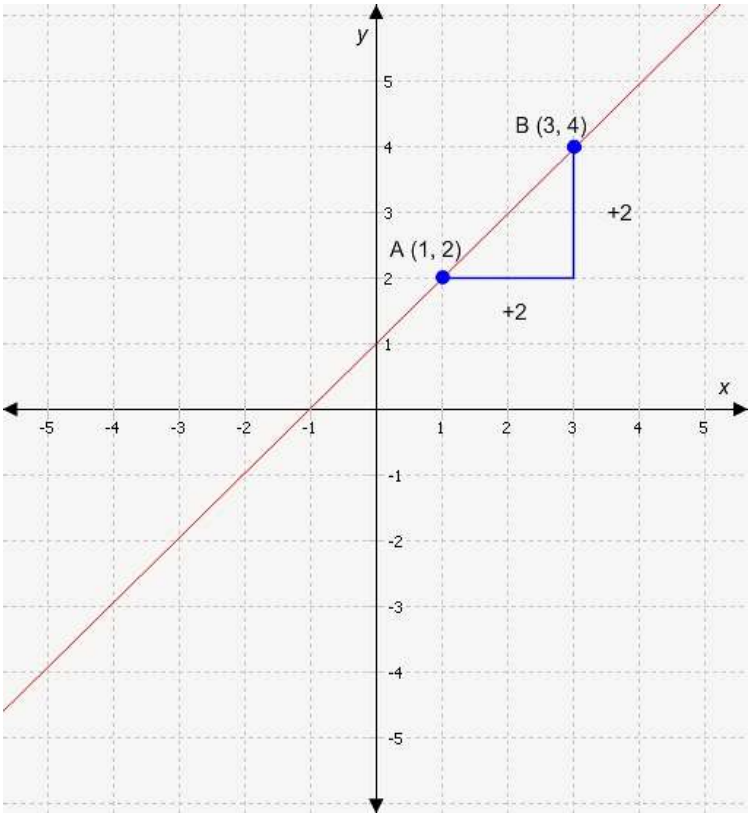
$$\text{rise} = y_2 - y_1$$

Run is the number of units we move left or right from one point to another. So, on a graph, run is the change in the x-values.

$$\text{run} = x_2 - x_1$$

Let's consider two points on the line: A(1, 2) and B(3, 4).

Slope is rise over run. So, the slope is $\frac{2}{2}$, or 1. Both the rise and run are positive, so the slope is also positive.



The sign of the slope of a line depends on the vertical direction of the line as it moves to the right.

- The slope of a line that moves *upward* as it moves to the right is positive.
- The slope of a line that moves *downward* as it moves to the right is negative.

Lesson Activity Negative Slope

A submarine is at a depth of 100 feet below sea level. Then it starts to dive at a rate of 50 feet per minute.

Question 1

A. For this situation, what is the independent variable (include units)?	**The independent variable is time in minutes.**
B. What is the dependent variable (include units)?	**The dependent variable is elevation in feet.**
C. What is the slope of the line that models this situation?	**Because the submarine is diving, the unit rate will be negative. So, the unit rate for this situation will be -50 feet per minute. Therefore, the slope of the line is -50.**
D. What is the *y*-intercept of the line that models this situation?	**The submarine is 100 feet below sea level before the dive. Because the submarine is below sea level, the value is negative. So, the y-intercept of the line for this situation is -100.**
Question 2 Write the equation of the line that models the situation using the slope and *y*-intercept found in parts C and D of question 1.	**y = -50x – 100**

Lines are continuous. They move in both directions without **stopping**. However, the full length of a line may not be suitable for a given situation. By reading the situation carefully, we can figure out which part of the line we need to consider.

Question The veterinarian told Sally that her cat needs to lose 5 pounds. A healthy weight loss for her cat would be about 1 pound every 2 weeks. To model this situation, she graphed the following line, where *y* represents the number of pounds left to lose and *x* represents time in weeks.

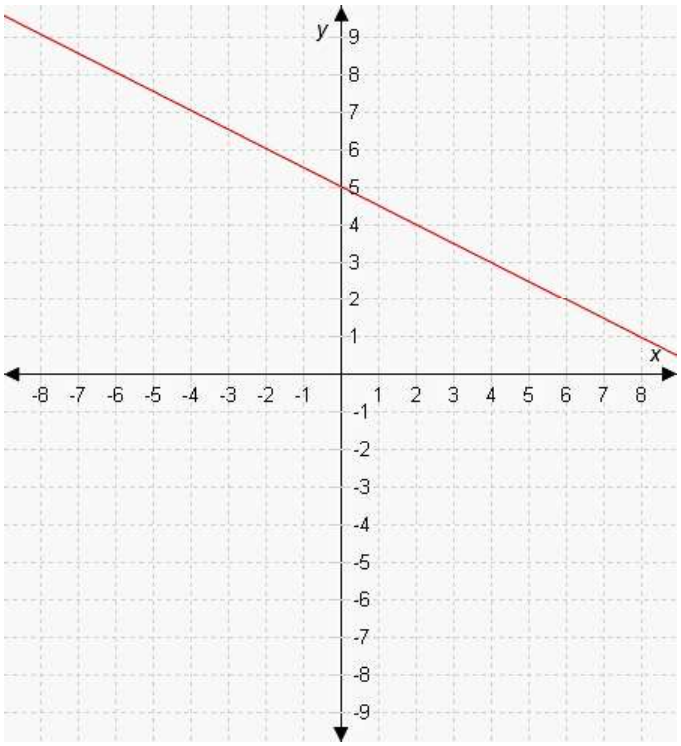
The unit rate is ○ $\frac{1}{2}$ ◉ $-\frac{1}{2}$ pound per week.

Because the number of pounds left for the cat to lose decreases each week, the rate is **negative**.

The *y*-intercept is **5**.

The equation for this line is $y =$ **-1/2** $x +$ **5**.

If she continues the line, she would see that it would take **10** weeks for her cat to lose 5 pounds.



Summary

When you create a linear equation that models a real-world situation, what are some important phrases to help determine the slope and *y*-intercept?

answers will vary