Name: ______________________________ Date: ______________

Circle Constructions

Objective

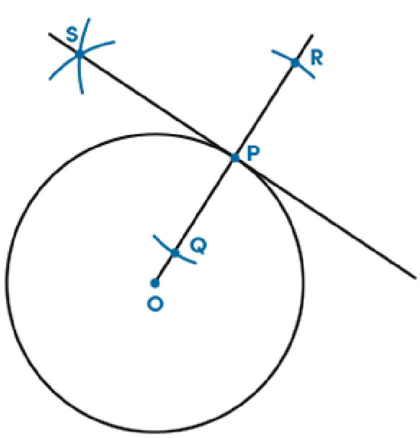
In this lesson, you will construct the inscribed and circumscribed circles of a triangle and a line tangent to a circle and prove properties of angles for a quadrilateral inscribed in a circle.

Tangent Lines

A tangent line is a line that intersects a circle at **exactly** **one** point on the circle. Tangent lines are also **perpendicular** to a radius of the circle.

Constructing a Tangent Line from a Point on the Circle

Construct a tangent line that intersects the circle at point P.



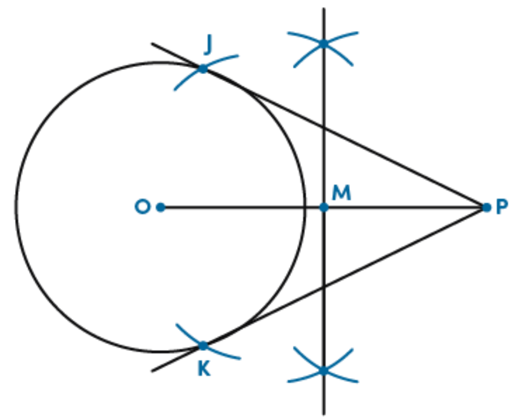
1. Draw a line from point **O** through point P, extending beyond point P.
2. With the compass on point **P** and set to a width less than ***OP***, draw an arc on each side of P (arcs Q and R).
3. With the compass on point **Q** and set to a width greater than ***QP***, draw an arc **outside** the circle on one side of point P.
4. Repeat step 3 with the compass on point **R** and draw another intersecting arc to create point S.
5. Draw a line through points **P** and S.

   Line **PS** is a tangent to the circle at point P.

Constructing a Tangent Line to a Point Off the Circle

Construct a tangent line to point P.

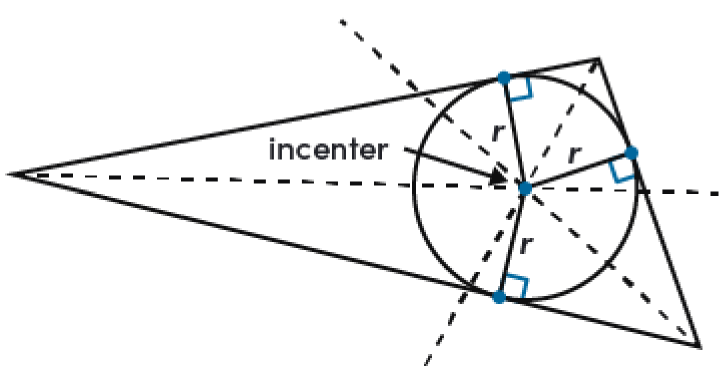
1. From center O, draw a line segment connecting points O and **P**.
2. Use a compass and straightedge to construct the **perpendicular** **bisector** for segment OP. Label the **midpoint** of segment OP as point M.
3. With the compass on point M and set to a width of ***OM***, draw arcs at the two places where the compass intersects the **circle** (J and K).
4. Draw lines JP and **KP**. These lines are **tangent** to circle O and intersect point **P**.



Inscribed Circle of a Triangle

An inscribed circle of a triangle is the **largest** circle that can be drawn inside a triangle, such that every side of the triangle is **tangent** to the circle.

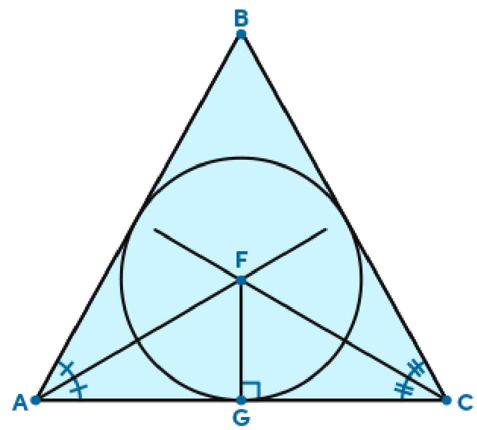
- The point where all the **angle** **bisectors** of the triangle meet is called the **incenter**. It coincides with the **center** of the inscribed circle.
- The incenter is located the same perpendicular distance to each **side** of the triangle.



We can use a compass and straightedge to construct the incenter and inscribed circle of a triangle.

Constructing an Inscribed Circle

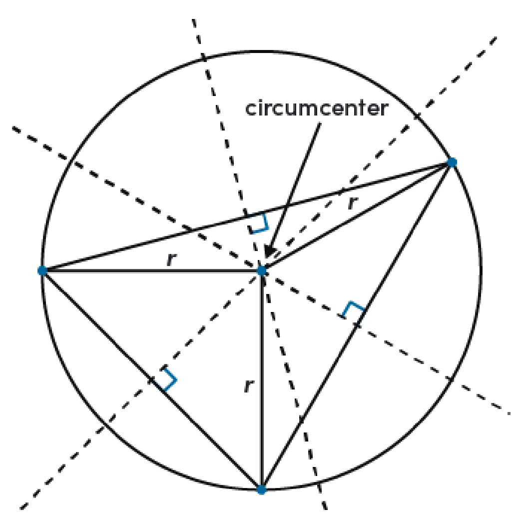
1. Construct two **angle** **bisectors** in the triangle. The intersection is the **incenter** (point F), which will be the center of the inscribed circle.



2. From the incenter, construct a segment **perpendicular** to one of the sides (segment FG). This will be the **radius** of the inscribed circle.
3. With the compass on the incenter (point F) and set to a width of the **radius**, draw a **circle**.

Circumscribed Circle of a Triangle

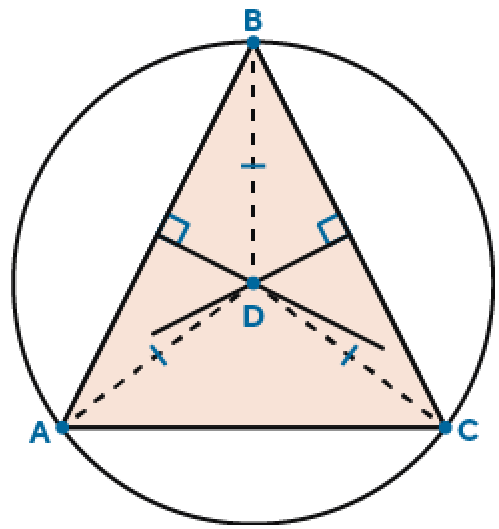
The circumscribed circle of a triangle is a circle that passes through **all** the **vertices** of the triangle.



- The point where all the **perpendicular** **bisectors** of the triangle meet is called the **circumcenter**. It coincides with the **center** of the circumscribed circle.
- Since the circumscribed circle passes through each **vertex** of the triangle, the **segments** between the circumcenter and each vertex are **radii** of the circumscribed circle.

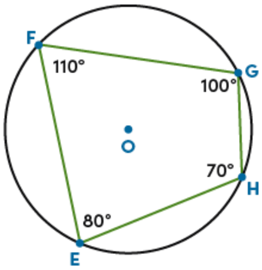
Constructing a Circumscribed Circle

1. Construct two **perpendicular** **bisectors** in the triangle. The **intersection** is the circumcenter (point D), which will be the **center** of the circumscribed circle.

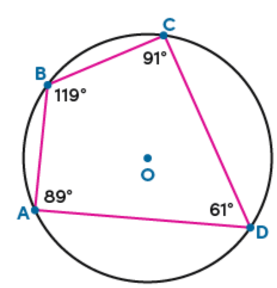


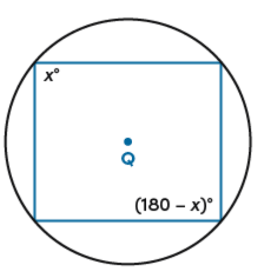
2. Set the compass width equal to the distance between the **circumcenter** and one of the **vertices** of the triangle, which is the **radius** of the circle.
3. Use the compass to draw the circle centered at the **circumcenter**.

Inscribed Quadrilateral of a Circle



- It is not always possible to inscribe or circumscribe a circle for polygons with more than **three** sides.
- **Opposite** angle measures of quadrilateral ABCD have a sum of **180**°, or are **supplementary**.
- If the opposite angles of a quadrilateral are not **supplementary**, it **cannot** be circumscribed by a circle.





- For a parallelogram's opposite angles to be both **congruent** and supplementary, the parallelogram must be a **rectangle**. So, rectangles are the only type of parallelogram that can be **inscribed** in a circle.