Constructing and Interpreting Functions

Objective

In this lesson, you will

construct and interpret functions given in verbal descriptions, two coordinate values, tables, or a graph.

Functions and Verbal Descriptions

To create a function from a verbal description:

1. Identify the independent (***x*-variable**) quantity and the dependent (***y*-variable**) quantity.
2. Determine the **rate** at which the ○ *x*-value ◉ *y*-value changes with respect to a change of one unit in the ◉ *x*-value ○ *y*-value.
   Find the rate of change.
3. Determine the **initial** value. Find the ○ *x*-value ◉ *y*-value when the ◉ *x*-value ○ *y*-value is zero.

The rate of change and the initial value can have a **positive** or a **negative** sign.

Example: A company owns a network of family campgrounds. The company charges an annual membership fee of $600 per member. In addition, each member must pay $5 per night in camping fees. What function represents the total fees that a member pays in year in terms of the number of nights camped?

First: Identify the **independent** and **dependent quantities**.

In this situation the number of nights a member stays is the ◉ independent ○ dependent quantity.

The fees paid are based on the number of nights stayed.

So, we represent these nights with the x-variable.

The total fees paid by the member are the ○ independent ◉ dependent quantity.

So, we represent them with the y-variable.

Second: Find the **rate of change** of the function.

The camping fee of $5 **per** night is the **change** in total fees a member pays per night of camping. So, 5 is the **rate** of **change** of the function.

Third: Determine the **initial value** of the function.

The annual membership fee of $600 corresponds to 0 nights of camping. So, **600** is the initial value of the function. Now let's substitute the **initial** value and the **rate** of change into the equation for a linear function, $y = mx + b$. The initial value, **b**, is 600, and the rate of change, **x**, is 5.

$$y = 5x + 600$$

Suppose we have a function and a situation. How can we interpret the function in terms of the situation?

First: We need to find the **independent** and **dependent** quantities in the situation.

Knowing the relationship of the two variables gives us a better understanding of what the function means.

Second: We note the function's rate of **change** and **initial** value.

The rate of change is the change in the **dependent** quantity with every change of **one** unit in the **independent** quantity.

The initial value is the **dependent** quantity's value when the **independent** quantity is zero.

Lesson Activity Interpreting a Function

Steven's cat drinks water from a bowl. The function represents the ounces of water left in the bowl in terms of hours since it was filled.

$$y = -\frac{1}{2}x + 4$$

A. Which variable represents the ounces of water left in the bowl?	*the y-variable*
B. Which variable represents the number of hours since the bowl was filled?	**the x-variable**
C. What does the 4 represent?	*the number of ounces of water in the bowl when first filled*
D. What does the value $-\frac{1}{2}$ represent?	**the change in ounces of water in the bowl with every hour**

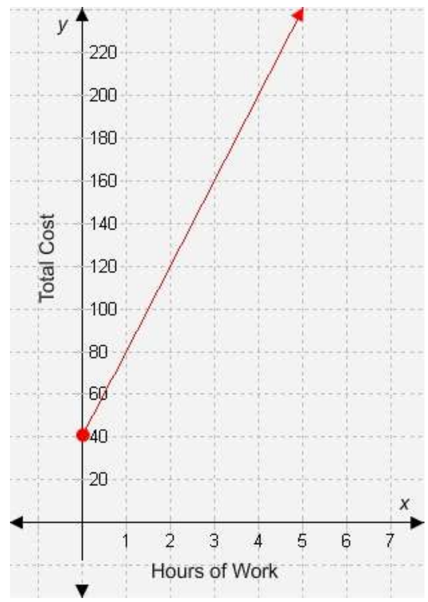
Functions, Graphs, and Points

We can also create the equation form of the function from a graph by determining the **variables** (x and y), **rate** of change, and **initial** value.

Lesson Activity Interpreting Graphs

Chris has a clogged drain. The graph represents how much it will cost Chris to get it fixed in terms of the number of hours the plumber works.

A. What is the initial value of the function?	The initial value or y-intercept is at **40**.
B. What does the initial value represent?	the cost of the service call (the cost of getting the plumber to Chris' house)
C. What is the rate of change of the function? Use the points (0, 40) and (1, 80).	slope = $\frac{\text{change in } y}{\text{change in } x} = \frac{80-40}{1-0} =$ **40**
D. What does the rate of change represent?	the cost for **each** **hour** the plumber works



Lesson Activity Function from Two Points

We can create a function if we know any two points that satisfy it.

A linear function passes through the points (-2, 3) and (-3, 5). Find the equation of the function.

Part A: What is the rate of change of the linear function that passes through the two points?

The rate of change is the **slope** of the line passing through the two points.

$$\text{rate of change (slope)} = \frac{\text{change in } y\text{-value}}{\text{change in } x\text{-value}} = \frac{5-3}{-3-(-2)} = \frac{2}{-1} = -2$$

Part B: What is the function's initial value, *b*?

Substitute the rate of change, *m*, found in Part A and the values of one of the ___**points**___ into the equation $y = mx + b$. Let's use the first point, (-2, 3).

__**5**__ = -2(__**-3**__) + *b*

Part C: Solve the equation in Part B to find the initial value, *b*.

$3 = -2(-2) + b$

3 = __**4**__ + *b*

__**-1**__ = *b*

Part D: Use the rate of change, __**-2**__, and the initial value, __**-1**__, to write the equation of the function.

y = __**-2**__*x* – __**1**__

Summary

Which method of finding the equation of a function do you prefer? Explain your reasoning using examples.

answers will vary