Equivalent Expressions

Objective

In this lesson, you will

rewrite expressions in different forms to show how quantities are related.

Understanding Equivalent Expressions

Two expressions are equivalent if either of them can be **rewritten** to create the other expression by applying the properties of operations and combining like terms.

- Equivalent expressions give the **same** **result** as the original expression for any given values of the variables.

Example: Let's check whether the expressions $\frac{3(x+4)}{2}$ and $\frac{3x}{2}+6$ are equivalent.

Transform the first expression to see if it gives us the second expression.

$$\frac{3(x+4)}{2} = \frac{3x+12}{2}$$ Apply the **Distributive** Property.

$$= \frac{3x}{2}+\frac{12}{2}$$

$$= \frac{3x}{2}+6$$ Divide.

The expressions are mathematically the **same**.

It's important to verify using more than one value of the variable, because two expressions might give the same result by chance. By using two or more variables, you increase your odds of proving them equivalent.

Example: Check for the equivalence of the two expressions using *x* = 1 and *x* = 2.

Expression	$x = 1$	$x = 2$
$\frac{3(x+4)}{2}$	$\frac{3(1+4)}{2} = \frac{15}{2} =$ 7.5	$\frac{3(2+4)}{2} = \frac{18}{2} =$ 9
$\frac{3x}{2} + 6$	$\frac{3(1)}{2} + 6 = 1.5 + 6 =$ 7.5	$\frac{3(2)}{2} + 6 = 3 + 6 =$ 9

Percent Increase and Percent Decrease

Equivalent expressions can help us relate two or more quantities in a word problem.

Example: Steve's favorite clothing store is having a weekend sale. He notices that the price of jeans has been marked down by 15%, so he buys two pairs. If the original price of each pair is *y* dollars, write an expression to show how much money Steve spent on jeans.

Original cost of each pair of jeans = __y__ dollars

The discount is 15% of __y__; or 0.15*y*

The discounted price of each pair is (●) *y* − 0.15*y* ( ) *y* + 0.15*y*.

Rewrite the expression by assuming a __coefficient__ of 1 on the first value of *y*.

$$y - 0.15y = 1y - 0.15y$$
$$= (1 - 0.15)y$$
$$= \underline{0.85}\, y$$

The intermediate expression we found shows that subtracting 0.15 of the price from the original price is the same as __multiplying__ the price by __0.85__.

So, *y* − 0.15*y* is the same as __0.85__ *y*. Since Steve bought two pairs of jeans, we multiply by 2 to arrive at the final answer: 2(0.85*y*) = __1.70__ *y*.

When working with percentages, be sure to distinguish between the phrase "increase (or decrease) by" and the phrase "increase (or decrease) to" in the problem. These two phrases do not mean the same thing.

Example: A property on York Street has an original value of *y*, and a nearby property on Zion has an original value of *z*. Write the increased value of each property in terms of *y* and *z*, respectively.

increases ***by*** 300%	increased ***to*** 300%
The current value, *y*, of the York Street property increases by 300%. The new value of the property is given by this expression: *y* + 300%(*y*). Simplify the expression and recognize that 300% is the same as $\frac{300}{100}$. $y + 300\%(y) = y + \frac{300}{100}y$ $= y + 3y$ $= \underline{4}\,y$ Increasing *by* 300% is the same as **multiplying** the original value by **4**.	The property on Zion increased to 300% of its original value, *z*. The new property value is 300% of *z*, or 300%(*z*). Simplify the expression, recognizing that 300% is the same as $\frac{300}{100}$. $300\%(z) = \frac{300}{100}z$ $= \underline{3}\,z$ Increasing to 300% is the same as **multiplying** the original value by **3**.

More Equivalent Expressions

Here's a situation involving multiple variables that we can model using equivalent expressions.

Example: At the souvenir shop, Theo buys *k* key chains and *p* photo frames where each chain and frame costs $2.49. He also buys a T-shirt for $5.99. What is the total cost of Theo's purchases in terms of *k* and *p*?

Represent the total cost in two ways:

1.) Find the cost of each item and add them up to find the total cost.

The cost of *k* key chains is **2.49** *k* dollars. The cost of *p* photo frames is **2.49** *p* dollars. The cost of one T-shirt is $**5.99**.

Total cost = **2.49** *k* + **2.49** *p* + **5.99**

2.) Another way to find the total cost is by multiplying $2.49 by the total number of key chains and photo frames (*k* + *p*) that Theo bought since both items have the same unit price. Then, complete the expression by adding the cost of the T-shirt, $**5.99**.

Total cost = **2.49**$(k + p)$ + **5.99**

✓ Check for equivalence of these two expressions by rewriting the first expression to match the second.

Use the Distributive Property to carry out the factorization.

2.49 k + **2.49** p + **5.99** = **2.49**$(k + p)$ + **5.99**

Remember that representing a scenario using different expressions does not change the result of the scenario as long as the expressions are **equivalent**.

Summary

How can you prove that two expressions are equivalent? Is there more than one way?

answers will vary