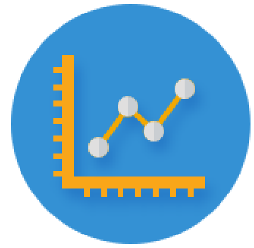
Name:______________________________ Date: ______________

Writing and Solving Exponential Equations



Objective

In this lesson, you will create exponential equations in one variable and use them to solve problems.

an exponential relationship: a relationship in which **a change in the independent variable results in a constant percentage rate of increase or decrease in the dependent variable**

Exponential Expressions and Equations

Exponential expressions contain a **constant** raised to a **power**.

called the "base"

called the "exponent"

The **exponent** states the number of times we multiply the **base** by itself.

Example: $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = \mathbf{32}$ ⟶ **2** is the base, and **5** is the exponent.

Equations that have a variable as an **exponent** are called exponential equations.

exponential growth	• an **increase** in a quantity at a constant percentage rate per unit interval • examples: a quantity that **doubles** every day, compounded interest,the human population
exponential decay	• a **decrease** in a quantity at a constant percentage rate per unit interval • examples: a quantity that **halves** each day, the depreciating value of a car,the weight of radioactive materials

GENERAL FORM OF EXPONENTIAL EXPRESSIONS

$$A(b)^x$$

- A: the **initial** amount or **starting** value
- x: the **exponent** or power of the expression
- b: the base, also called the **factor**

- When $x = 0$, the expression will equal ***A*** .
- If ***b*** > **1**, the expression will model exponential growth.
- If ***b*** is between **0** and **1**, the expression will model exponential decay.

The growth or decay rate, r, is typically written as a **percentage**.

If r is a rate of growth, r = ***b*** **–1** .	If r is a rate of decay, r =**1** – ***b*** .

Question

Expression	Growth or Decay	Rate
$30(1.25)^x$	**growth**	r = **1.25** – **1** = **0.25** = **25** %
$500(0.75)^x$	**decay**	r = **1** – **0.75** = **0.25** = **25** %
$2(2)^x$	**growth**	r = **2** – **1** = **1** = **100** %

REWRITING THE BASE

To solve exponential equations, we have to rewrite the bases so they are **equal** and the equation is in the form $b^x = b^y$.

If the bases of an exponential equation are equal, then the **exponents** are equal as well: $x = y$.

Example:

$3^{x+1} = 81$

$3^{x+1} = 3^4$

x + **1** = **4**

x = **3**

Rewrite the right side of the equation so that it has the same base as the left side, 3.

Variations of this problem type include:

- working with bases in the form of $\left(\frac{1}{b}\right)^x$, which we rewrite as b **-x**.
- rewriting both sides of the equation with a new base
- isolating the exponential term

Question

$$4^{5x} = \left(\frac{1}{32}\right)^{1-x}$$

$$(2^2)^{5x} = \left(\frac{1}{2^5}\right)^{1-x}$$

$$(2^2)^{5x} = (2^{-5})^{1-x}$$

$$2^{10x} = 2^{-5+5x}$$

$$10x = -5 + 5x$$

5x = -5

Rewrite both sides of the equation so that they have the same **base** of **2**.

Modeling with Exponential Equations

We can use an exponential equation to model a relationship between two variables, but must determine the initial value, the base, and the exponent.

Lesson Activity Question

Question 2

A. Dr. Steve is working with a new radioactive substance in his lab.
He currently has 64 grams of the substance and knows it decreases at a rate of 25% every hour.

The **initial amount** is **64**.

The situation represents exponential **decay**.

The rate, r, is **0.25**.

B. Dr. Steve needs to calculate how long it will take his 64-gram sample of a radioactive substance to decay to 27 grams.
To write the expression to model the amount of radioactive substance remaining, substitute for A and b in the general form of an exponential expression.

⇒ A = **64**

⇒ b = **0.75** (to find b, substitute **0.25** for r in the equation $r = 1 - b$ and solve.)

⇒ $A(b)^t$ = **64 (0.75)**^{t}

The value of this expression needs to equal the remaining amount, 27 grams.

C.

$$27 = 64(0.75)^t$$

$$\left(\frac{27}{64}\right) = (0.75)^t$$

$$\left(\frac{27}{64}\right) = \left(\frac{3}{4}\right)^t$$

$$\left(\frac{3}{4}\right)^3 = \left(\frac{3}{4}\right)^t$$

$$\underline{\ 3\ } = t$$

The substance will decrease from 64 grams to 27 grams after **3** hours.

For problems involving interest that is not compounded annually, this formula can be used:

$$P = P_0\left(1 + \frac{r}{n}\right)^{nt}$$

- P_0 is the **initial deposit.**
- ***r*** is the interest rate.
- ***n*** is the number of times per year that the interest is compounded.
- *t* is the time, in years.

Example: $300 is placed in account where it earns 4% interest. How much will be in the account after 5 years if the interest is compounded annually? What if it was compounded quarterly?	**4 % Compounded Annually**	**4 % Compounded Quarterly**
	$A(b)^x$ = **300** (**1.04**)^5 ≈ **365**	$P = 300\left(1 + \frac{0.04}{4}\right)^{(4)(5)}$ = 300(**1.01**)^{20} = **366.06**