

Name: \_\_\_\_\_ Date: \_\_\_\_\_

# Cross Sections of Three-Dimensional Objects



## Objective

In this lesson, you will describe two-dimensional figures that result from slicing three-dimensional figures.

## Understanding Cross Sections of Three-Dimensional Objects

Cross sections are formed when a three-dimensional object is sliced by a geometric plane.

Cross sections of three-dimensional objects are two-dimensional shapes of various sizes.



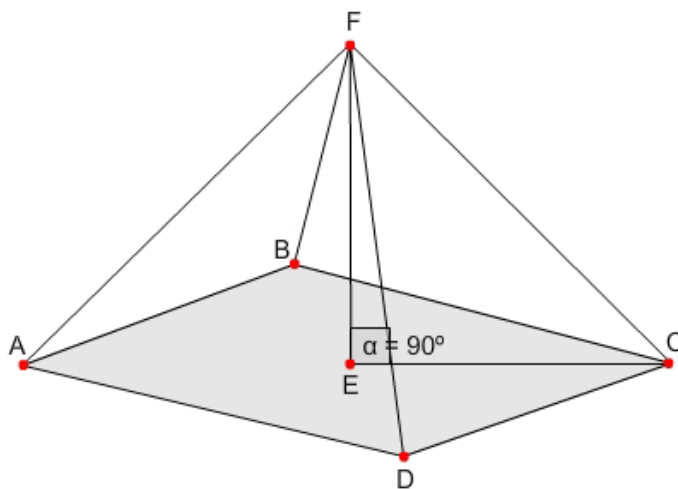
If the object has  $n$  faces, the plane can intersect at most  $n$  faces.

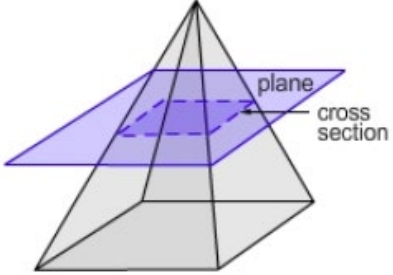
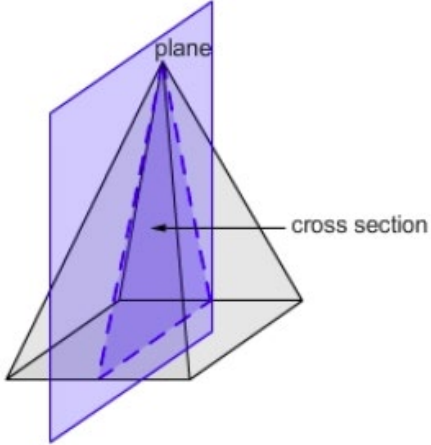
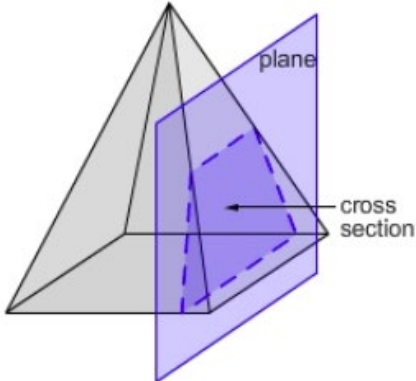
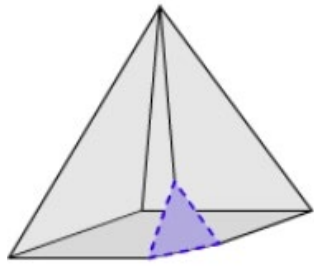
So, a cross section may have any number of sides less than or equal to  $n$ .

## CROSS SECTIONS OF A RIGHT RECTANGULAR PYRAMID

A pyramid is a three-dimensional object whose base is a polygon and faces are triangles.

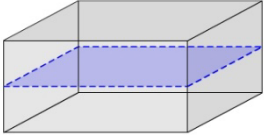
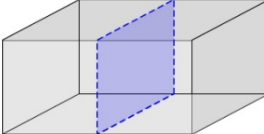
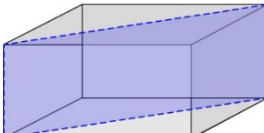
A right rectangular pyramid is a pyramid whose base is a rectangle and top vertex is directly above the center of the base.



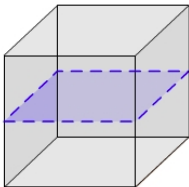
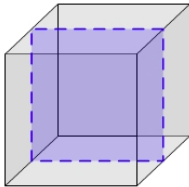
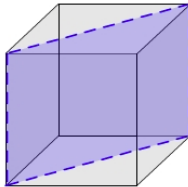
Slicing Plane	Cross Section	
parallel to the base	<ul style="list-style-type: none"> <li>• same <b>shape</b> as the base but not the same <b>size</b></li> <li>• As the cross section moves closer to the pyramid's base, its size <b>increases</b>.</li> </ul>	
perpendicular to the base, passing through the top vertex	<p><b>triangle</b></p>	
perpendicular to the base, not passing through the top vertex	<p><b>trapezoid</b></p>	
<p><b>cutting across a corner</b></p>	<p>triangle</p>	

## CROSS SECTIONS OF A RIGHT RECTANGULAR PRISM

**prism:** a six-sided figure with opposite sides that are parallel and of the same shape and size. Rectangular prisms have rectangular bases and rectangular faces that are perpendicular to the bases. A cube is a rectangular prism with all sides of the same length.

	<p>A cross section parallel to the rectangular base is congruent to the rectangular base.</p>
	<p>A cross section <u>perpendicular</u> to the rectangular base is also a rectangle and congruent to a pair of faces.</p>
	<p>A cross section along the diagonals of opposite faces is a <u>rectangle</u>.</p>

**Question** What is the shape of each cross section?

 <p data-bbox="311 1480 540 1549">square</p>	 <p data-bbox="699 1480 928 1549">square</p>	 <p data-bbox="1079 1480 1308 1549">rectangle</p>
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**MORE CROSS SECTIONS**

Cross sections can be taken from any three-dimensional figure.

**Lesson Activity**

Identify the shapes of the cross sections formed.

**Question 1:**

**Part A:** Describe the shapes formed when particular planes pass through **cones**.

Description of Plane	Cross Section
plane parallel to the circular base, not passing through the tip of the cone	a circle
plane parallel to the circular base, passing through the tip of the cone	a point
plane not parallel to the base, not passing through the base, and making an angle that is less than that made by the slant height of the cone	an oval
plane making an angle that is greater than that made by the slant height, passing through the tip of the cone	an isosceles triangle

**Question 2:**

**Part A:** Describe the shapes formed when particular planes pass through **cylinders**.

Description of Plane	Cross Section
plane parallel to the vertical axis	a rectangle
plane parallel to the circular base	a circle
plane making an angle with the vertical axis without passing through the base or top surface	an oval
plane making an angle with the vertical axis and passing through either the base or the top surface, but not both	a cut-off section of an oval

**Question 3:**

**Part A:** Describe the shapes formed when particular planes pass through **hexagon-based pyramids**.

Description of Plane	Cross Section
plane parallel to the base	<b>a hexagon</b>
plane passing through exactly two sides and the base	<b>a triangle</b>
plane passing through exactly three sides and the base	<b>a quadrilateral</b>
plane perpendicular to the base and passing through the vertical axis	<b>an isosceles triangle</b>

**Question 4:**

**Part A:** Describe the shapes formed when particular planes pass through **pentagonal prisms**.

Description of Plane	Cross Section
plane parallel to the vertical axis	<b>a rectangle</b>
plane parallel to the base	<b>a pentagon</b>
plane making an angle with the vertical axis without passing through the base or the top surface	<b>a pentagon</b>
plane making a sharp angle with the vertical axis and passing through the base and top surface	a pentagon, a hexagon, or a heptagon
other planes at various angles with the vertical axis	a pentagon, a quadrilateral, or a triangle

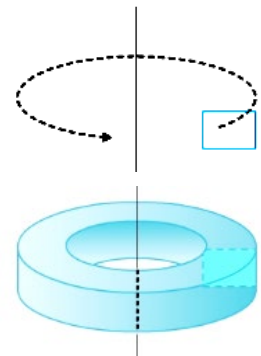


If a corner is sliced off a pyramid or prism, the resulting cross section will have a minimum of 3 sides.

## Rotating Two-Dimensional Objects

If a shape on a plane is rotated around an axis, not perpendicular to the plane of the object, the region of space swept out by the rotation is a **three**-dimensional object called a solid of **revolution**.

The line of rotation is sometimes called the **axis** of rotation.



A right triangle is rotated about an axis that passes through its height as shown. Which type of object will be formed by its rotation?

The rotation forms a **cone**.