KEY

Recognizing Proportional Relationships

Objective

In this lesson, you will

decide whether two quantities are in a proportional relationship.

Introduction

Proportional relationships are part of daily life, especially in activities such as cooking, shopping, and banking.

- Two quantities are said to have a proportional relationship if their ratios are **equivalent**.
  - → Chefs use proportional relationships in **recipes** so that each serving is the same, no matter how many servings they make.
- If two quantities are **proportional** and you know one quantity, then you can find the other.
  - → By knowing the **unit** price for a product, for example, you can find the **total** cost for any number of units.

Proportional Relationships

Jenna is baking oatmeal raisin cookies. The recipe calls for a half cup of raisins for a batch of 12 cookies. Jenna can bake more than 12 cookies if she **proportionally** increases the amount of every ingredient, like the raisins.

The table shows different amounts of raisins and the **corresponding** numbers of cookies Jenna can bake.

Cups of Raisins	Number of Cookies
$\frac{1}{2}$	12
1	24
$1\frac{1}{2}$	36

Ratio of the number of cups of raisins to the number of cookies: the ratio of the first pair of values in the table is $\frac{\frac{1}{2}}{12}$.

Ratio of the number of cups of raisins to the number of cookies: the ratio of the second pair of values is $\frac{1}{24}$.

There is a proportional relationship between the number of cups of raisins and the number of cookies that Jenna can prepare. All the **ratios** of the pairs in the table are **equivalent**.

Cups of Raisins	Number of Cookies
$\frac{1}{2}$	*12*
1	*24*
$1\frac{1}{2}$	**36**

Any number times 1 is equal to the number itself.

The second ratio from the table, $\frac{1}{24}$, is **equivalent** to the first ratio, $\frac{\frac{1}{2}}{12}$.

→ To prove them equivalent, we need to find a fraction equivalent of **1** that we can multiply by $\frac{1}{24}$ to get $\frac{\frac{1}{2}}{12}$:

→ To get from the ratio $\frac{1}{24}$ to the ratio $\frac{\frac{1}{2}}{12}$ we can multiply the numerator of $\frac{1}{24}$ by $\frac{1}{2}$. The fraction equivalent of 1 that has $\frac{1}{2}$ as the numerator also has **2** as the **denominator**.

$$\frac{1}{24} \times \frac{\frac{1}{2}}{\frac{1}{2}} = \frac{\frac{1}{2}}{12}$$

❖ This same process can be used to show that other proportions from the table are also equivalent.

Leslie baked carrot muffins and apple muffins for a school event. She made five more carrot muffins than apple muffins in each batch she baked. The table shows the numbers of carrot muffins and apple muffins in each of three batches.

Batch	Apple Muffins	Carrot Muffins
1	*10*	**15**
2	**15**	*20*
3	*20*	*25*

First Batch: $\frac{number\ of\ apple\ muffins}{number\ of\ carrot\ muffins} = \frac{10}{\boxed{15}}$

Second Batch: $\frac{number\ of\ apple\ muffins}{number\ of\ carrot\ muffins} = \frac{15}{\boxed{20}}$

The relationship between the number of apple muffins and the number of carrot muffins is proportional only if the ratios of the numbers of muffins from each batch are equivalent.

→ Look for a fraction equivalent of **1** that we can multiply by $\frac{15}{20}$ to get $\frac{10}{15}$.

→ To get from the ratio $\frac{15}{20}$ to the ratio $\frac{10}{15}$, multiply the **numerator** of $\frac{15}{20}$ by $\frac{2}{3}$. The fraction equivalent of 1 that has $\frac{2}{3}$ as the numerator also has $\frac{2}{3}$ as the **denominator**, so we also multiply the denominator of $\frac{15}{20}$ by $\frac{2}{3}$.

$$\frac{\boxed{15}}{20} \times \frac{\frac{2}{3}}{\frac{2}{3}} = \frac{10}{\frac{40}{3}} \neq \frac{\boxed{10}}{15}$$

→ The ratios $\frac{10}{15}$ and $\frac{15}{20}$ are **not** equivalent, so the numbers of apple muffins and carrot muffins in the table do not have a proportional relationship.

Recognizing Proportional Relationships from Tables

Proportional relationships are often displayed in tables. If you're given a table with pairs of values for two quantities, you can calculate the ratio for each pair. If the ratios are equivalent, you know the two quantities have a proportional relationship

Which table shows a proportional relationship between x and y?

X	Y
1	5
2	6
3	7

X	Y
2	3
4	6
6	9

X	Y
4	16
8	64
12	144

X	Y
1	3
2	9
3	27

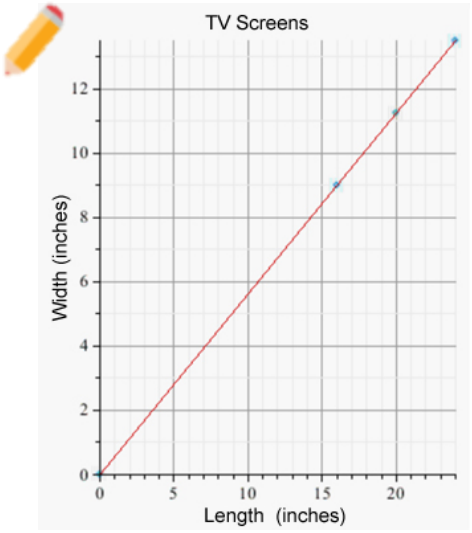
Check whether the ratios of the x and y values in each row of the table are equivalent. For the second table, the equivalent ratio (x:y) is 2:3. In other words, $\frac{2}{3} = \frac{4}{6} = \frac{6}{9}$.

Recognizing Proportional Relationships from Graphs

Another method of checking for a proportional relationship between two quantities is to graph their relationship. If the graph is a straight line that passes through the origin, the relationship is proportional.

The table gives the lengths and widths (in inches) of three different sizes of TV screens.

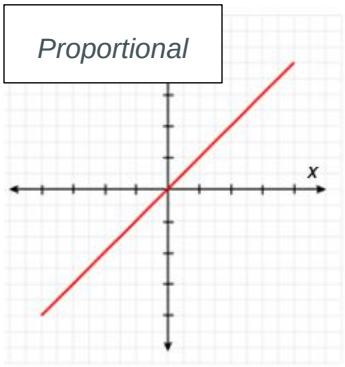
Length (inches)	Width (inches)
16	*9*
20	
	13.5



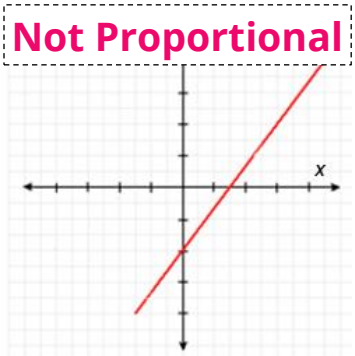
The relationship is proportional because the graph is a straight line that passes through the **origin**.

A graph of a proportional relationship is always a **straight** **line** that passes through the origin. In a **nonproportional** relationship, the line might be straight; but such a line won't pass through the origin.

Proportional

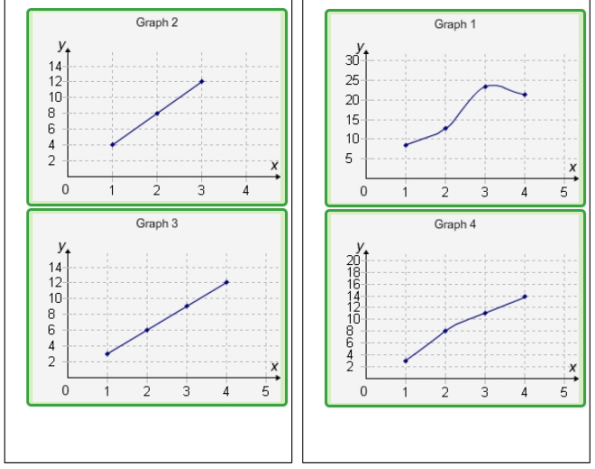


Not Proportional



Proportional

Not Proportional



Summary

How can tables and graphs be used to determine if values are proportional?

Answers will vary.