Name: ______________________ Date: ______________

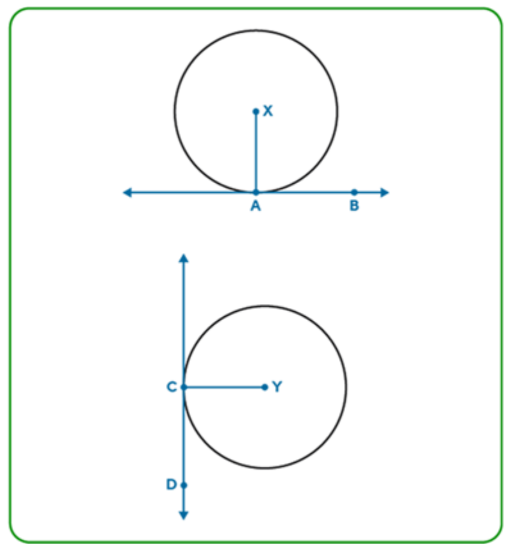
Circle Relationships

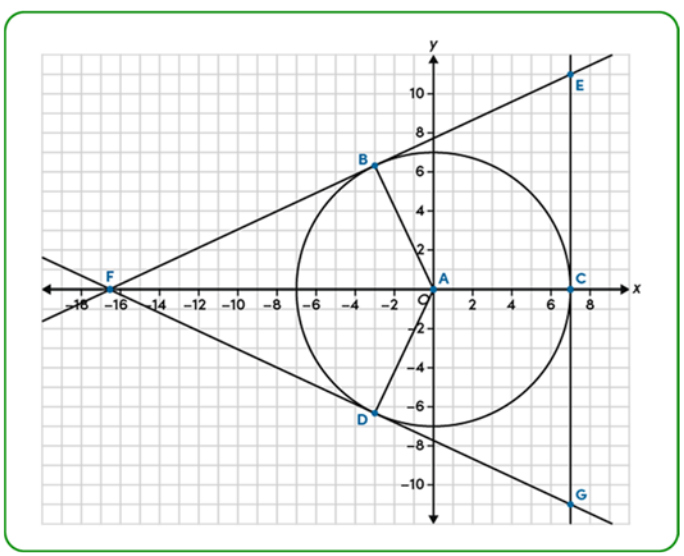
Objective

In this lesson, you will investigate relationships between lines, segments, and angles in circles.

Radius of a Circle and Tangent Lines

- A line tangent to a circle at its extreme top or bottom point is **horizontal**________.
- A line tangent to a circle at its far left or right point is **vertical**______.

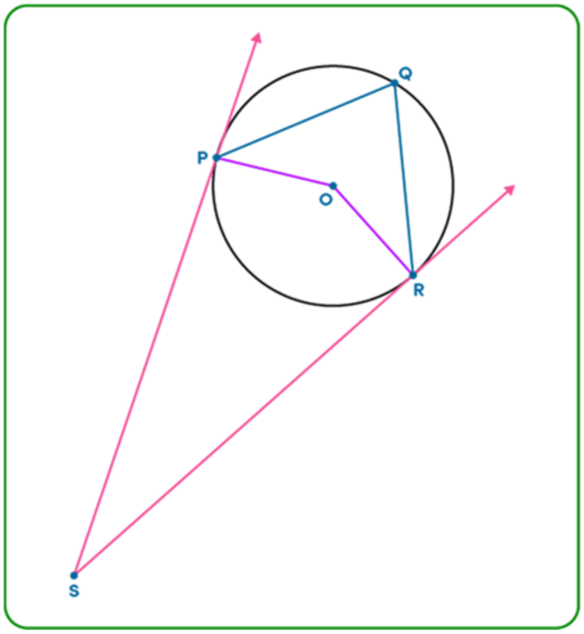




- The radius of a circle is always **perpendicular**________ to the tangent at the point where the radius and tangent **intersect**______.
- Perpendicular lines have slopes that are **opposite**_____ **reciprocals**________.

  So, their slopes have a product of **-1**____.

Central, Inscribed, and Circumscribed Angles



- central angle: an angle with the center of the circle as its **vertex**____ and two **radii**____ as its sides
- inscribed angle: an angle with its vertex on the **circle**____ and its sides formed by two **chords**____ or a **tangent**____ and a chord
- circumscribed angle: an angle with its vertex **outside**____ a circle and its sides formed by two **tangents**____
- ∠POR is a **central**____ angle, ∠**PQR**____ is an inscribed angle, and ∠PSR is a **circumscribed**____ angle.

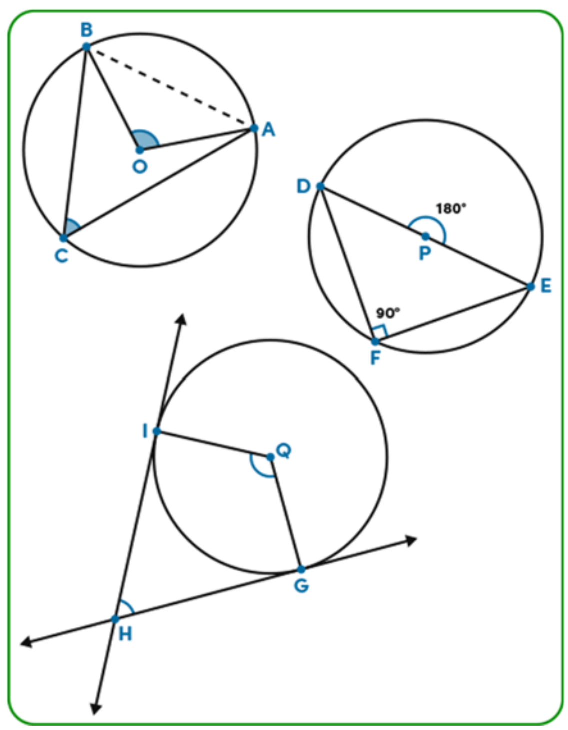
1. If an inscribed angle and a central angle intercept the **same** **arc**____, then the measure of the inscribed angle is **half**____ that of the central angle.

$m\angle ACB = \frac{1}{2}(m\angle AOB)$

2. If the chord bound by the inscribed and central angles is a diameter, the central angle measures **180**____°, and any inscribed angle bound by the diameter is **90**____°.

3. If a circumscribed angle and a central angle **intercept**____ the same arc, then the angles are **supplementary**____.

$m\angle GHI + m\angle GQI =$ **180**____°



Chords

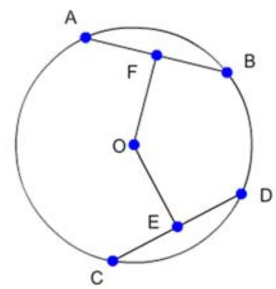
Properties of Chords

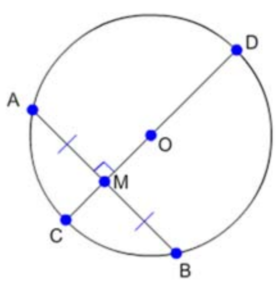
Congruent chords of a circle are equidistant from the **center**, and they intercept congruent **arcs**.

If $\overline{AB} \cong \overline{CD}$, then $\overline{OE} \cong$ **$\overline{OF}$**, and if $\overline{AB} \cong \overline{CD}$, then $\overset{\frown}{AB} \cong$ **$\overset{\frown}{CD}$**

The **converse** is also true:

If $\overline{OE} \cong \overline{OF}$, then $\overline{AB} \cong$ **$\overline{CD}$**, and if $\overset{\frown}{AB} \cong \overset{\frown}{CD}$, then $\overline{AB} \cong$ **$\overline{CD}$**





A diameter perpendicular to a chord **bisects** the chord, and a diameter that bisects a chord is also **perpendicular** to the chord.

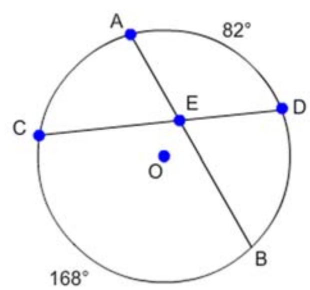
If $\overline{AB} \perp \overline{CD}$, then $\overline{AM} \cong$ **$\overline{BM}$**, and if $\overline{AM} \cong \overline{BM}$, then $\overline{CD} \perp$ **$\overline{AB}$**

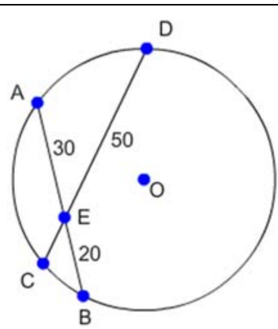
The perpendicular bisector of any chord passes through the **center** of the circle.

When two chords intersect, the measure of an angle between them is equal to **half** the sum of the measures of the **intercepted** arcs of that angle and that angle's **vertical** opposite.

$m\angle AED = \frac{1}{2}(m\overset{\frown}{CD} + m\overset{\frown}{AB})$

$m\angle AED =$ **125**°





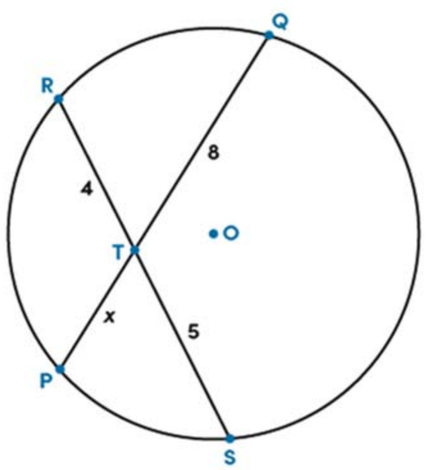
When two chords intersect, the **products** of the lengths of the line segments on each chord are the same.

(**AE**)(EB) = (CE)(**ED**)

12 = CE

Finding Lengths of Intersecting Chords

Chords RS and PQ intersect within circle O at point T. Find the length of segment PT.



- When two chords intersect, the products of the lengths of the line segments on **each** **chord** are **equal**.
- So, $(PT)(\underline{TQ}) = (\underline{RT})(TS)$.

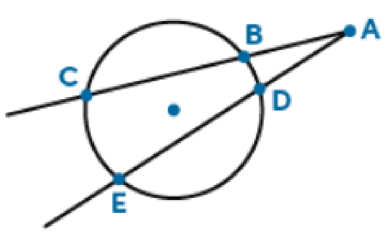
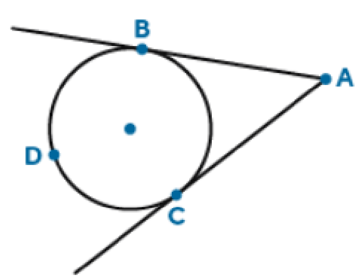
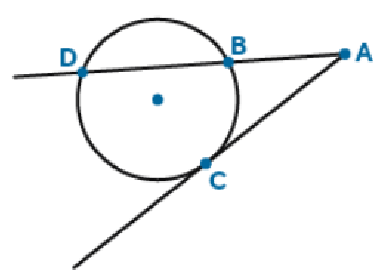
$$x(\underline{8}) = (\underline{4})(5)$$

$$x = \frac{5}{2}$$

Tangents and Secants

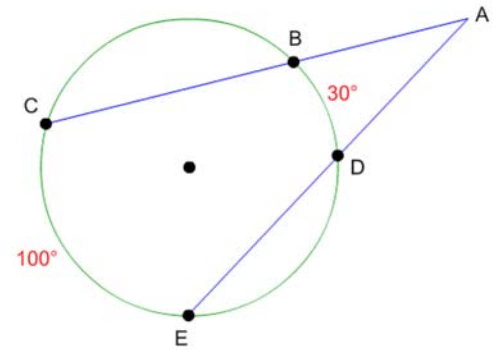
Angles Formed Outside the Circle

When tangents or secants intersect outside the circle, the measure of the angle formed by those segments is equal to **half** the **difference** of the measures of the two arcs that are intercepted by the sides of the angle.

two secants	two tangents	secant and tangent
		
$m\angle A = \frac{1}{2}(m\overset{\frown}{CE} - m\overset{\frown}{BD})$	$m\angle A = \frac{1}{2}(m\overset{\frown}{BDC} - m\overset{\frown}{BC})$ $m\overset{\frown}{BDC} = (\underline{360} - m\overset{\frown}{BC})$	$m\angle A = \frac{1}{2}(m\overset{\frown}{DC} - m\overset{\frown}{BC})$

Finding an Angle Measure

Angle A is formed by two secants.



$m\angle A = \frac{1}{2}(m\overset{\frown}{CE} - m\overset{\frown}{BD})$

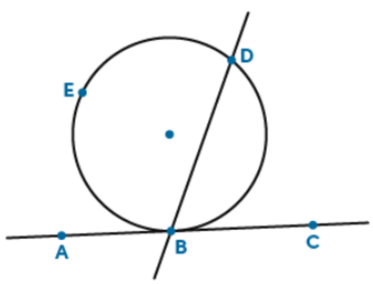
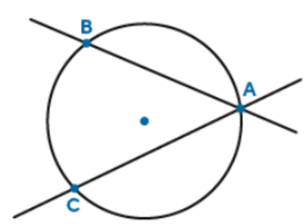
$= \frac{1}{2}(100° - 30°)$

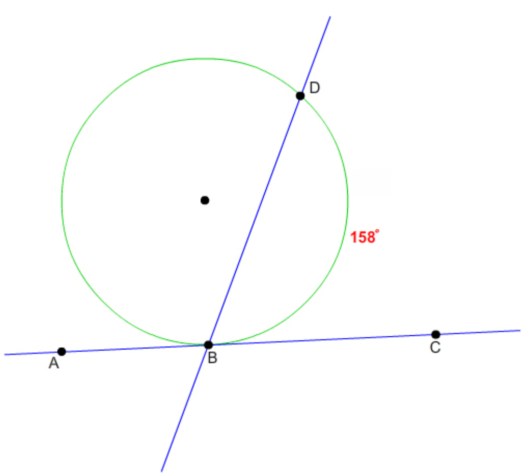
$= 35°$

Angles Formed on the Circle

When secants, chords, or tangents intersect on a circle, an **inscribed** angle is formed.

The measure of an **inscribed** angle is equal to **half** the measure of the intercepted **arc**.

secant and tangent	two secants
	
$m\angle DBC = \frac{1}{2}(m\overset{\frown}{DB})$ $m\angle DBA = \frac{1}{2}(m\overset{\frown}{DEB})$	$m\angle BAC = \frac{1}{2}(m\overset{\frown}{BC})$



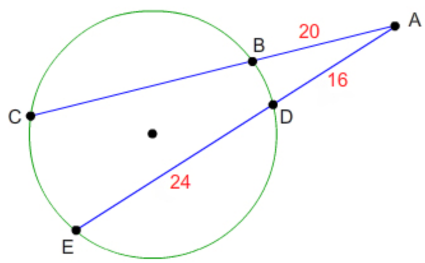
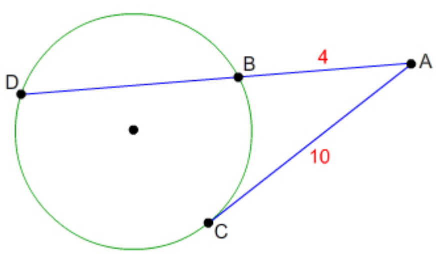
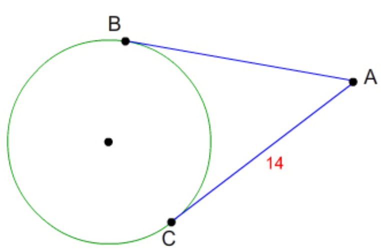
Angle DBC is formed by the intersection of a secant and tangent.

$m\angle A = \frac{1}{2}(m\overset{\frown}{BD})$

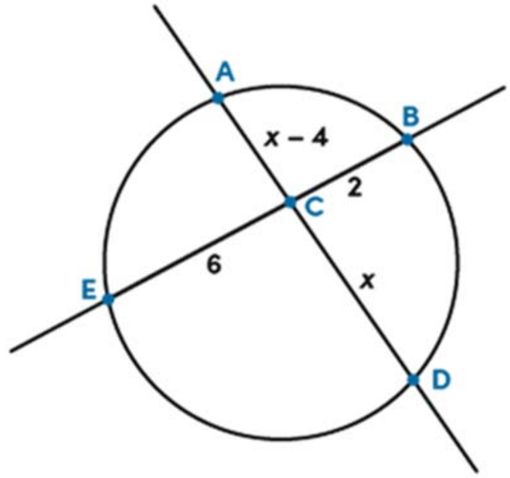
$= \frac{1}{2}(158)°$

$= 79°$

Pieces of Secants and Tangents

Two Secants  	The products of the distances to both the near and far points of intersection with the circle, along each secant, are **equal**. (***AB***)(*AC*) = (***AD***)(***AE***) (**20**)(*AC*) = (**16**)(**40**) *AC* = **32**
Secant and Tangent  	The product of the distances to both the near and far points of intersection with the circle, along the secant, is equal to the **square** of the length of the **tangent** segment. (***AB***)(*AD*) = (***AC***)^2 (**4**)(*AD*) = (**10**)^2 *AD* = **25** *BD* = **25** – **4** = **21**
Two Tangents  	When two tangents intersect, the point of intersection is the **same** **distance** from both points of tangency. AB = AC = **14**

Find the lengths of $\overline{AC}$ and $\overline{CB}$.



(*AC*)(**CD**) = (**BC**)(**CE**)

(**x - 4**)(**x**) = (**2**)(**6**)

x^2 – **4** x = **12**

This is a **quadratic** equation. Rewrite equal to **zero** and solve by **factoring**.

x^2 – **4** x – **12** = **0**

(x – **6**)(x + **2**) = 0

x = **6** or x = **-2**

Since segment lengths are not **negative**, CD = **6** and AC = **2**.