Name: ______________________________ Date: ____________________

Symmetry

Objective

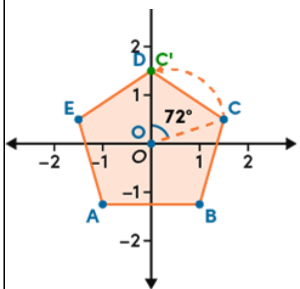
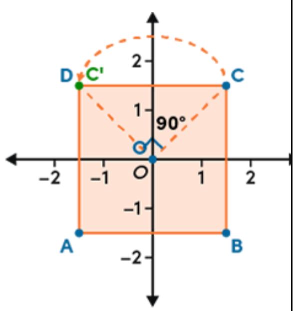
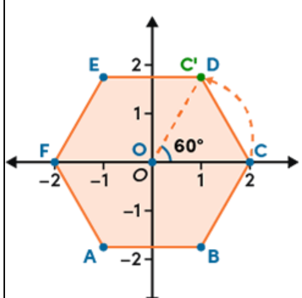
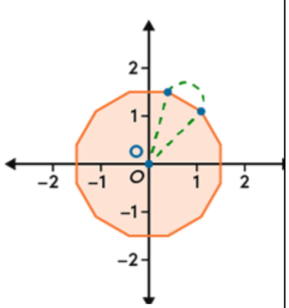
In this lesson, you will describe the rotations and reflections that carry a given polygon onto itself.

Rotations

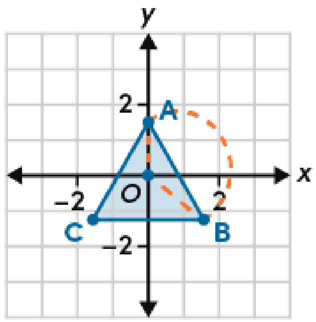
ROTATION SYMMETRY

- When rotating a figure **less** than a full revolution of **360**° results in it mapping back onto itself, the figure has **rotational symmetry**.
- The number of times that the figure maps back onto **itself** in one **360**° revolution is called the **order** of **rotational symmetry**.
  - For a **regular** polygon, the **order** of rotational symmetry equals the polygon's number of **sides**, n. This is because each side of a **regular** polygon **subtends** an angle of $\frac{360°}{n}$ at the center.
- The angle of **symmetry** is the smallest angle by which a figure can be **rotated** to map back on **itself**.
  - For a **regular** polygon, a rotation through an angle of $\frac{360°}{n}$ about the **center** maps one vertex exactly onto the next vertex in the **preimage**. So, $\frac{360°}{n}$ is the **angle** of symmetry for a **regular** polygon.

REGULAR POLYGONS

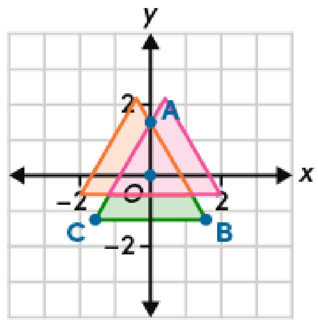
Polygon name	Diagram	Order of rotational symmetry	Angle of Symmetry (degrees)	Maps onto itself when rotated by (degrees)
regular pentagon		**n = 5**	72	**72, 144, 216, 288, 360**
square (**regular quadrilateral**)		n = 4	**90**	**90, 180, 270, 360**
regular hexagon		**n = 6**	**60**	**60, 120, 180, 240, 300, 360**
regular dodecagon		**n = 12**	**30**	30, 60, 60, 120, 150, 180, 210, 240, 270, 300, 330, 360

ROTATION OF TRIANGLES



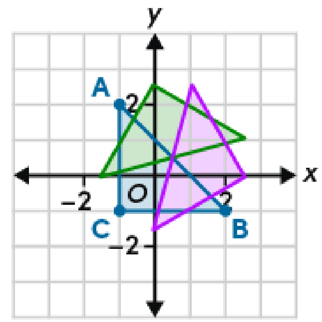
Equilateral triangle:

- **three** congruent sides and angles
- is a **regular polygon**
- order of **rotational** symmetry *n* = **3**
- angle of **symmetry** of **120**°
- maps onto itself when **rotated** by **120**°, **240**°, and **360**°



Isosceles triangle:

- has **two** congruent sides and angles
- **is not** a regular polygon
- does not map onto itself if **120**° or **240**° about its **center**
- only maps onto itself after a complete **360**° revolution
- **does not** have rotational symmetry



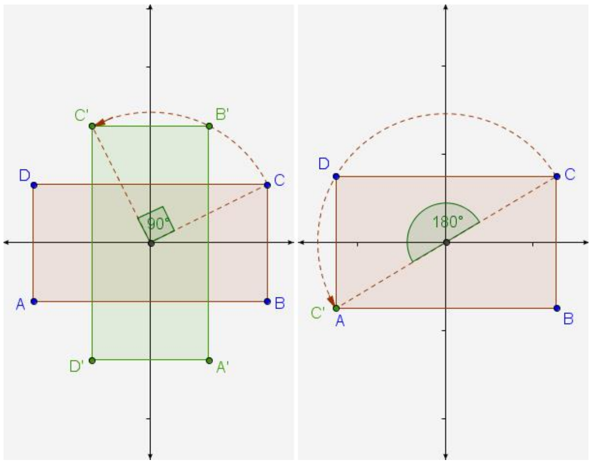
Right triangle:

- at most **two** congruent sides and angles
- **is not** a regular polygon
- does not map onto itself if rotated **120**° or **240**° about its **center**.
- only maps onto itself after a complete **360**° revolution
- **does not** have rotational symmetry

IRREGULAR POLYGONS

Some **irregular** polygons do have **rotational** symmetry.

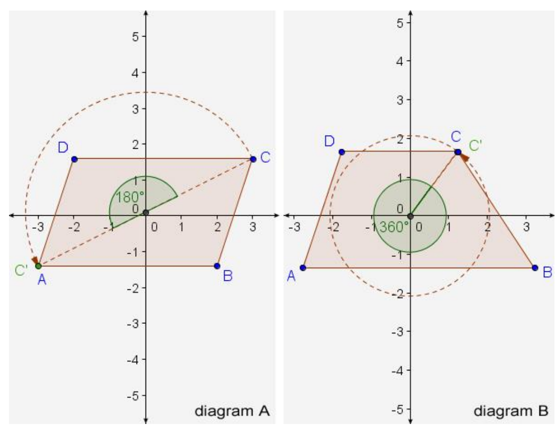
- Even though it also has four sides, a **rectangle** does not have the full symmetry of a **square**.



  - When we rotate a rectangle counterclockwise about its **center**, the rotated rectangle coincides with the **preimage** only at **180**° and **360**°.
  - The order of **rotational** symmetry is **2**, and the **angle** of symmetry is **180**°.

- Like a rectangle, a **parallelogram** also maps onto itself only **twice** during one complete **rotation** about its **center**.

- A trapezoid has only **one** pair of **opposite** sides that are **parallel**. The **nonparallel** sides may or may not be **equal** in **length**.
  - This **lack** of **symmetry** does not allow the shape to map onto itself more than once when it is **rotated** about its **center**.

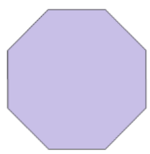


POINT SYMMETRY

When a figure can be mapped onto itself by **rotating** the figure about a **center** point, it is said to have **point symmetry**.

- The figure looks the same when **upside down**. Any point on the figure has a matching point the same **distance** from the point of symmetry, but in the **opposite** direction.
- Point symmetry occurs in any figure with **rotational symmetry** of order **2**, or a multiple of **2**.

ROTATIONAL SYMMETRY

Type of Figure	Example	Rotational Symmetry?	Point Symmetry?
Regular polygon		Yes	**Yes**
Irregular polygon		**Yes**	No
Recycling icon		**Yes**	**No**
Playing card		**Yes**	**Yes**
Triskelion		**Yes**	**No**

Reflections

Rotations______ about certain angles **map**____ a polygon back onto itself, **reflections**_____ about certain **lines**_____ do the same.

LINE SYMMETRY

When a **line**___ of **reflection**_____ maps a figure back **onto**____ **itself**____, it can also be considered a **line**___ of **symmetry**_____.

- Line symmetry exists when **two**___ halves of a figure are **mirror**____ **images**____ of each other. One **half**____ is a **reflection**____ of the other **half**____.
- When a **line**___ of **reflection**_____ is also a **line**___ of **symmetry**_____, it passes through the **center**_____ of the figure.

Figure		Lines of symmetry
Triangles	**scalene**______	0
	isosceles	**1**
	equilateral	**3**
Quadrilaterals	parallelogram and **trapezoid**___	**0**
	isosceles trapezoid and **kite**___	1
	rectangle____ and rhombus	**2**
	square	**4**

- **Regular**___ polygons have all **sides**___ congruent and all **angles**___ congruent. A **regular**___ polygon with **n**___ sides has **n**___ **lines**___ of symmetry.

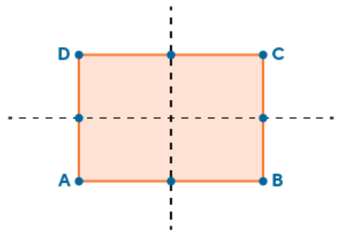
- **Line**___ symmetry can be observed in **block**___ letters.
  - The block letter **A**___ has **1**___ line of symmetry.
  - The block letters **H**___ and **O**___ have **2**___ lines of symmetry.

LINES OF REFLECTIONS FOR REGULAR POLYGONS

A **regular**___ polygon has ***n***___ lines of reflection.

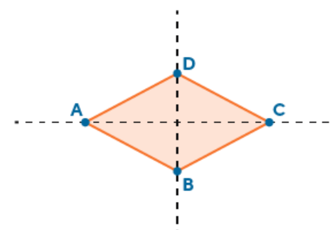
- If *n* is even:
  - $\frac{n}{2}$ of the lines are **perpendicular**___ **bisectors**___ joining the **midpoints**___ of the **opposite**___ sides.
  - $\frac{n}{2}$ are **angle**___ **bisectors**___ joining the **opposite**___ vertices.
- If *n* is odd:
  - The polygon **doesn't**___ have pairs of vertices that are **opposite**___ one another. Instead, each vertex is **opposite**___ the **midpoint**___ of the **opposite**___ side.
  - The **perpendicular**___ **bisectors**___ of the sides represent the only **lines**___ of **reflection**___ that will flip the shape back onto itself. Since each side of the polygon has a unique **perpendicular**___ **bisector**___, the polygon still has ***n***___ **lines**___ of **reflection**___.

REFLECTIONS OF IRREGULAR POLYGONS

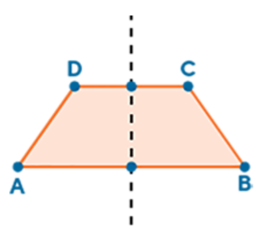


- A rectangle has **2** lines of symmetry. A rectangle is symmetric only about the **perpendicular** **bisectors** of its pairs of **opposite** sides.

- Some parallelograms **cannot** map back to themselves across any **line** of **reflection**.
  - If all the **sides** of a parallelogram are **equal** in length, then the **diagonals** of the parallelogram **bisect** each other at **right** angles. Such a parallelogram has **2** lines of reflection and can be reflected onto itself across the **diagonals** because the **angles** and **sides** directly **opposite** one another are congruent.



- If the **sides** of a trapezoid are all **different** lengths, there is **no** line across which a **reflection** coincides with the **preimage**.



  - If a trapezoid has **nonparallel** sides **equal** in **length**, then the **parallel** sides have a common perpendicular **bisector**. **Isosceles** trapezoids have **1** line of reflection.