Name: ______________________ Date: ______________

Variation Equations

Objective

In this lesson, you will represent and solve problems involving direct and inverse variation.

Direct Variation

- when the **quotient** of two variables is constant and 0 is paired with **0**
- may be represented by an equation in either the form $y = kx$ or $\frac{y}{x} = k$, where $k \neq 0$

The constant, *k*, is called the constant of **variation**.

- If *y* varies directly as an *n*th **power** of *x*, then $y = kx^n$ or $\frac{y}{x^n} = k$.

Question

Can these aspects of the Dylan's vacation can be expected to vary directly?

Aspect	Vary Directly?
the gallons of gasoline used and the number of miles driven (Driving twice as many miles will use twice as much gas.)	**Yes**
the volume of a circular hotel swimming pool and the square of its radius (In the equation for the volume of a cylinder, $V = \pi h r^2$, doubling the value of r^2 will **double** the value of *V*.)	Yes
the number of attractions visited in a day and the time spent at each one (Doubling the time spent at attractions will **not** result in being able to visit twice as many.)	**No**
the number of pins that can be bought for $20 and the price per pin (When the price of the pins is doubled, the number that can be purchased is **halved**.)	No

Consider a situation where walnuts are sold at a rate of \$2.50 per pound.
The total cost will always equal 2.5 times the pounds of the walnuts purchased.

We can say the total cost of walnuts **varies** **directly** as the number of pounds being purchased and model this situation with an equation in the form $y = kx$.

→ When y is the total cost and **x** is the weight of the walnuts purchased, **k** is the price per pound.

→ The total cost is \$**0** when 0 pounds are purchased.
For all other amounts purchased, the **ratio** of the total cost to the amount purchased is constant.

$\frac{5}{2} =$ **2.5** $\frac{10}{4} =$ **2.5** $\frac{15}{6} =$ **2.5**

$y =$ **2.5** x

x	$y = 2.5x$
0	2.5(0) = 0
2	2.5(2) = 5
4	2.5(4) = 10
6	2.5(6) = 15

What we see in the nut scenario generalizes to any situation involving **direct** variation, replacing x or y with the nth power of the variable when necessary:

- We say **y** varies directly as **x**.
- The situation can be represented by an equation in the form $y = kx$ or $\frac{y}{x} =$ **k**, or a straight line graphed on a coordinate plane.
- When one variable is equal to **0**, the other variable is also equal to **0**.
- The variable y changes at a **constant** rate, k, with respect to the other variable, x.

Example:

Consider a carpet that costs \$2.29 per square foot.

We can say that the cost, y, of carpet for a square bedroom varies directly with the **square** of the bedroom's length, in feet, x.

y $= 2.29$ **x** ^{2} or $\frac{y}{x^2} =$ **2.29**

Question

In this table, y varies **directly** as x.

To identify the table where y varies directly as x,

we look for a table where $y = 0$ when $x =$ **0** and

the ratio $\frac{y}{x}$ is the **same** for all the ordered pairs in the table.

x	y
0	0
2	12
4	24
6	36

We can use an equation representing a direct variation scenario to solve problems.

Example: Tom's boss pays him an hourly wage. He earns $120 when he works 8 hours. How much Tom will be paid if he works 35 hours next week?

Because Tom receives an hourly wage, his **total** **earnings** vary directly as the hours he works.

We set up an equation and solve for k, the constant of variation.

y = total earning, x = hours worked

$$\frac{y}{x} = k$$

$$\frac{\textbf{120} \text{ dollars}}{8 \text{ hours}} = k$$

15 hours worked $= k$

After substituting 15 for **k** and 35 for **x** in the equation, solve for **y** to find his total earnings.

Tom will earn $**525** if he works 35 hours next week.

Using $\frac{y}{x} = k$

$$\frac{y}{35} = 15$$

$$\frac{y}{35} \cdot 35 = 15 \cdot 35$$

$$y = \textbf{525}$$

Using $y = kx$

$$y = 15 \cdot \textbf{35}$$

$$y = 525$$

Inverse Variation

- Two variables vary inversely when their **product** is constant.
- An inverse variation may be represented by either $xy = k$ or $y = \frac{k}{x}$ where $k \neq$ **0**.
- As with direct variation, **k** is the constant of variation and either of the variables can be raised to a **power**, if necessary.

Example: A grounds manager knows it takes 80 hours to clear the leaves from a campus.

Changing the number of workers will change the time required to clear the campus by a **reciprocal** of that factor.

The product of the number of workers and the hours spent clearing will always equal **80**.

→ The time to required clear the leaves varies **inversely** as the number of workers.

We model this with an equation in the form $y = \frac{k}{x}$.

→ When **y** is the total time to complete the job and **x** is the number of workers, **k** is 80 work hours. So, $y = \frac{80}{x}$.

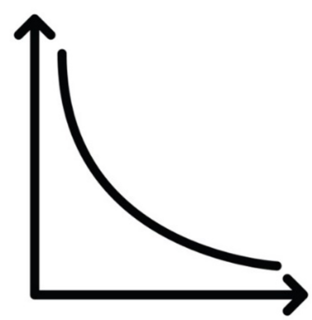
→ *y* is **undefined** when *x* = 0. Similarly, there is no value of *x* for which *y* = **0**.

x	$y = \frac{80}{x}$
0	undefined
4	$\frac{80}{4} =$ **20**
8	$\frac{80}{8} = 10$
16	$\frac{80}{16} = 5$

4 · 20, 8 · 10, and 16 · 5 all are equal to **80**

What we see in the leaf-clearing scenario generalizes to any situation involving inverse variation, replacing *x* or *y* with the *n*th power of *x* or *y* when necessary:

- We say **y** varies inversely as **x**.
- The situation can be represented by an equation in the form $xy = k$ or $y = \frac{k}{x}$.
- Neither variable can be equal to **0**.
- When one variable increases, the other **decreases**, and their product, *k*, is constant.



Question

The cube of *m* varies inversely as the square root of *n*.

Because the cube of *m* varies inversely as the square root of *n*, the **product** of m^3 and $\sqrt{n}$ is constant. These two equations model this relationship.

1. $m^3 n^{\frac{1}{2}} = k$

   We can use a **fraction** exponent for the square root and model the relationship.

2. $m^3 = \dfrac{k}{\sqrt{n}}$

   We can **isolate** the m^3 term and use a radical sign for the square root.

When we recognize inverse variation, we can use an equation to model the situation.

Example: A farmer has enough seed to feed 25 chickens for 24 days.
How many days the seed will last if the farmer must feed 30 chickens?

Because increasing the number of chickens will **decrease** the number of days the seeds lasts, the number of days varies **inversely** as the number of chickens. We set up an equation and solve for *k*, the constant of variation.	$y = \text{days}, x = \text{chickens}$ $xy = k$ $25 \cdot 24 = k$ $600 = k$	
After substituting **600** for *k* and **30** for *x* into an equation representing the situation, solve for *y*. The farmer's seed will last **20** days if she's feeding 30 chickens.	**Using $xy = k$** $30y = 600$ $y = 20$	**Using $y = \frac{k}{x}$** $y = \dfrac{600}{30}$ $y = 20$

Question

The variable p varies inversely as the square of q and when $p = 36$, $q = 25$.

Since p varies inversely as the square of q, this situation can be modeled by this equation:

$$p\underline{q}^2 = \underline{k}.$$

Substitute 36 for p and 25 for q, and solve for k, the constant of variation: $36 \cdot 252 = k$ $\underline{22{,}500} = k$ So, for all combinations of p and q, $pq^2 = \underline{22{,}500}$.	We can substitute 4 for p and solve for q: $\underline{4}q^2 = 22{,}500$ $q^2 = 5{,}625$ $q = \underline{75}$ When $p = 4$, $q = \underline{75}$.	We can substitute 10 for q and solve for p: $p \cdot 10^2 = 22{,}500$ $\underline{100}p = 22{,}500$ $p = \underline{225}$ When $p = 10$, $q = \underline{225}$.