Real Numbers

Objective

In this lesson, you will

use decimal expansion to understand the real number system.

Rational Numbers

Rational numbers are numbers that we can represent in the form $\frac{a}{b}$.

In this form, *a* and *b* are **integers** and *b* does not equal **0**.

We can also express all rational numbers as **decimal** numbers. The decimal form of a rational number either **terminates** or repeats.

Terminating Decimal:	Repeating Decimal:
$\frac{3}{8} = 0.375$	$\frac{1}{3} = 0.\overline{3}$

Every terminating or repeating decimal number can be expressed as a fraction of two **integers**.

Lesson Activity

Converting Decimal Numbers to Fractions

Question 2

Part A Repeating decimal numbers are more difficult to convert into fractions. If $x = 0.\overline{3}$, what does 10*x* equal?	$10x = 0.\overline{3} \cdot$ **10** $= 3.\overline{3}$
Part B What would happen if you subtracted *x* from 10*x*?	$10x - x = 3.\overline{3} - 0.\overline{3}$ **9**$x = 3$
Part C If you solve for *x*, what is the result?	$x = \frac{3}{9}$ $= \frac{1}{3}$
Part D What is the fraction that is equal to the repeating decimal number $0.\overline{3}$?	$x = 0.\overline{3}$ and $x = \frac{1}{3}$ So, $0.\overline{3} = \frac{1}{3}$.

Question

What fraction is equal to $0.\overline{13}$?

Let $x = 0.\overline{13}$.

$$100x - x = 13.\overline{13} - 0.\overline{13}$$

$$99x = \underline{13}$$

$$x = \frac{13}{99}$$

Irrational Square Roots

There are some real numbers that we can't express as terminating or repeating decimals. We call such numbers irrational numbers.

The most common irrational numbers are roots of non-perfect squares and cubes.

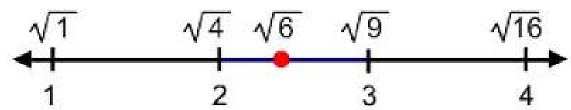
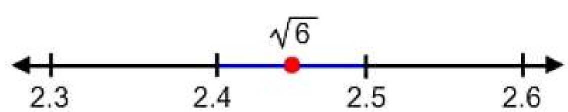
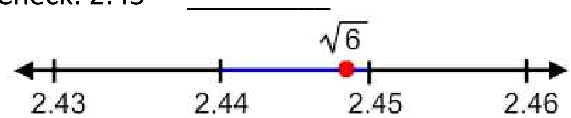
Take, for example, $\sqrt{2} = 1.41421356237309504$ .... We can write only the approximate value of the irrational number.

$$\sqrt{2} \approx 1.414$$

To find an approximate value of an irrational number, remember:

- Given two numbers *a* and *b*, if *a* > *b*, then a^2 > b^2.
- Given two numbers *a* and *b*, if *a* > *b*, then $\sqrt{a}$ > $\sqrt{b}$.

Example: Find the value of $\sqrt{6}$ up to the hundredth place.

Find two consecutive perfect squares between which the number $\sqrt{6}$ lies.	
Estimate the value of $\sqrt{6}$ using the guess-and-check method. We'll guess 2.4 first.	Guess: 2.4 Check: 2.4^2 = 5.76 
Since 2.4^2 is less than 6, increase the guess to 2.45. The result is slightly more than 6, but getting closer.	Guess: 2.45 Check: 2.45^2 = 6.0025 
The square of our next guess, 2.44, is 5.9536. This is farther from 6 than our last guess. So, $\sqrt{6} \approx$ 2.45.	Guess: 2.44 Check: 2.44^2 = 5.9536

Decimal Expansion of Irrational Numbers

Activity

Warren is tiling his bathroom. He wants to cut three different sizes of square tiles. Tile A must be 5 square inches, tile B must be 10 square inches, and tile C must be 15 square inches. He needs to know the side length of each tile.

Part A
Find the length of tile A, which has an area of 5 square inches. Estimate the value to five decimal places.

- Area of tile A = 5 square inches
- Side length of tile A = $\sqrt{5}$ inches
- Since $\sqrt{4} < \sqrt{5} < \sqrt{9}$, 2 < $\sqrt{5}$ < 3.

Guess	Check
2.2	**4.84**
2.23	*4.9729*
2.236	*4.999696*
2.23606	**4.9999643236**
2.23607	*5.0000090449*

The length of tile A should be approximately **2.23607** inches.

Part B
Find the length of tile B, which has an area of 10 square inches. Estimate the value to five decimal places.

- Area of tile B = 10 square inches
- Side length of tile B = $\sqrt{10}$ inches
- Since $\sqrt{9} < \sqrt{10} < \sqrt{16}$, 3 < $\sqrt{10}$ < 4.

Guess	Check
3.2	*10.24*
3.1	**9.61**
3.15	*9.9225*
3.16	*9.9856*
3.165	**10.017225**
3.1625	*10.00140625*
3.16227	*9.999951529*
3.16228	**10.0000147984**

The length of tile B should be approximately **3.16228** inches.

Part C
Find the length of tile C, which has an area of 15 square inches. Estimate the value to five decimal places.

- Area of tile C = 15 square inches
- Side length of tile C = $\sqrt{15}$ inches
- Since $\sqrt{9} < \sqrt{15} < \sqrt{16}$, **3** < $\sqrt{15}$ < **4**.

Guess	Check
3.8	*14.44*
3.85	**14.8225**
3.87	**14.9769**
3.875	*15.015625*
3.872	**14.992384**
3.8729	*14.99935441*
3.87299	*15.0000515401*
3.87928	*14.9999740804*

The length of tile C should be approximately **3.87928** inches.

Lesson Activity

Estimating from Decimal Expansions

Activity

Warren's tape measure is marked in increments of $\frac{1}{16}$ of an inch, not in decimal numbers. To measure the tiles, he needs to know the side lengths to the nearest $\frac{1}{16}$ of an inch.

Question 1

Part A
The length of tile A is about 2.23607 inches. Which mark on the tape measure is closest to 2.23607 inches?

The value 2.23607 lies after the whole number **2**. Divide the decimal part by $\frac{1}{16}$. That gives the number of sixteenths in 0.23607. Round to the nearest whole number.

$$0.23607 \div \frac{1}{16} = 0.23607 \cdot \mathbf{16}$$
$$= 3.77712 \text{ divisions}$$
$$\approx \mathbf{4} \text{ divisions}$$

Therefore, 2.23607 inches ≈ $2\frac{\mathbf{4}}{16}$ inches, or $2\frac{\mathbf{1}}{4}$ inches.

Part B The length of tile B is about 3.16228 inches. Which mark on the tape measure is closest to 3.16228 inches?	The value 3.16228 lies after the whole number 3. Divide the decimal part by $\frac{1}{16}$. That gives the number of sixteenths in 0.16228. Round to the nearest whole number. $0.16228 \div \frac{1}{16} = 0.16228 \cdot 16$ $= 2.59648$ divisions ≈ 3 divisions Therefore, 3.16228 inches $\approx 3\frac{3}{16}$ inches.
Part C The length of tile C is about 3.87298 inches. Which mark on the tape measure is closest to 3.87298 inches?	The value 3.87298 lies after the whole number 3. Divide the decimal part by $\frac{1}{16}$. That gives the number of sixteenths in 0.87298. Round to the nearest whole number. $0.87298 \div \frac{1}{16} = 0.87298 \cdot 16$ $= 13.96768$ divisions ≈ 14 divisions Therefore, 3.87298 inches $\approx 3\frac{14}{16}$ inches, or $3\frac{7}{8}$ inches.

Because irrational numbers don't terminate or repeat, we ○ can ◉ cannot find exact decimal expansions for them. Set the precision needed based on the situation.

Summary

Explain how you can convert a decimal that terminates after three decimal places to a fraction.

answers will vary