Name: ______________________ Date: ______________

Multiplying and Dividing Rational Expressions

Objective

In this lesson, you will multiply and divide rational expressions.

Simplifying Rational Expressions

A rational expression is the **ratio** of two **polynomial** expressions.

Terms are related to each other by **addition** or **subtraction**.

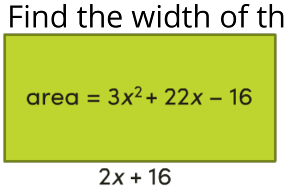
Rational expressions can have **variables** in their denominators. This means we need to watch out for **excluded** variable values, which are not allowed because they make the denominator **equal** to 0.

A rational expression is in **simplest** form when the numerator and denominator don't have any **common** **factors**.

Example 1: Simplify the expression Factor 2 from the numerator and rewrite the denominator as a product.	$\frac{2x+6}{x^2+x-6} = \frac{\boxed{2}(x+3)}{(\boxed{x}+3)(x-\boxed{2})}$
Cancel out the common factor (**$x + 3$**) in the numerator and the denominator.	$= \frac{2}{x-2}$

Example 2

Factor the GCF __4__ from the numerator and the GCF __2__ from the denominator.	$\frac{4x^2 - 12x - 16}{2x^3 - 8x^2 + 6x - 24} = \frac{4(x^2 - 3x - 4)}{2(x^3 - 4x^2 + 3x - 8)}$
Factor the trinomial in the numerator. Rewrite the denominator by grouping by a common factor.	$\frac{4(\boxed{x} - 4)(x + \boxed{1})}{2[x^2(x - \quad) + \quad (x - 4)]}$
Write the denominator as a __product__ of factors.	$\frac{4(x-4)(x+1)}{2[(x-4)(x^2+3)]}$
Cancel out the common factor in the numerator and the denominator, and simplify.	$\frac{\boxed{2}(x+1)}{(x^2 + \boxed{3})}$

Example: Find the width of the rectangle.  __Divide__ the area by the __length__.	width $= \frac{3x^2 + 22x - 16}{2x + 16}$
First, identify any excluded values.	excluded values: $2x + 16 =$ __0__ __2x__ $= -16$ $x = -2$
Now __factor__ the numerator and __cancel__ any common factors.	$\frac{(3x-2)(x+8)}{2(x+8)} = \frac{3x-2}{2}$ $= \frac{3}{2}x - 1$, where $x \neq$ __−8__

Multiplying Rational Expressions

To multiply rational expressions, or fractions, multiply **numerators** and the **denominators** of each expression. When you have a product, simplify it.

$$\frac{2x}{3x^2}\cdot\frac{x^2}{4}=\frac{2x\cdot x^2}{3x^2\cdot 4}=\frac{2\cdot x\cdot x\cdot x}{3\cdot x\cdot x\cdot 2\cdot 2}=\frac{x}{6}$$

It is important to consider excluded values when multiplying rational expressions.

Question

Identify any excluded values and rewrite the product in simplest form.

$$\frac{6y^2+18y-60}{3y^2-12y}\cdot\frac{y^2-16}{y^2+2y-8}$$

1. Identify excluded values.	Expression 1: $3y^2-12y=0$ **3y** $(y-$ **4** $)=0$ **3y** $=0$ and $y-4=$ **0** $y=$ **0** $\quad y=$ **4** Expression 2: $y^2+2y-8=0$ $(y-$ **2** $)(y+$ **4** $)=0$ $y-2=$ **0** and $y+$ **4** $=0$ $y=$ **2** $\quad y=$ **–4**
2. Factor all numerators and denominators.	$\frac{6(y+5)(y-2)}{3y(y-4)}\cdot\frac{(y-4)(y+4)}{(y-2)(y+4)}$
3. Multiply and cancel and common factors.	$\frac{2(y+5)}{y}=\frac{2y+10}{y}$

Dividing Rational Expressions

Just as when dividing rational numbers, to divide one rational expression by another first rewrite the division as **multiplication** by the **reciprocal** of the divisor.

Invert the **divisor** fraction and multiply.	$\frac{x^2-x-2}{x^2+x-2} \div \frac{x^2+2x+1}{2x^2-2x} = \frac{x^2-x-2}{x^2+x-2} \cdot \frac{2x^2-2x}{x^2+2x+1}$
Factor the quadratic expressions in the numerator and the denominator of each expression.	$= \frac{(x+1)(\boxed{x-2})}{(\boxed{x+2})(x-1)} \cdot \frac{\boxed{2x}(x-1)}{(x+1)(\boxed{x+1})}$
Combine the factors to rewrite them as a single **fraction**.	$= \frac{2x(x+1)(x-2)(x-1)}{(x+2)(x-1)(x+1)(x+1)}$
Cancel out common factors to get the simplest form.	$= \frac{\boxed{2x}(x-2)}{(x+2)(\boxed{x+1})}$

Rational expressions follow the **closure** property. Multiplying and dividing rational expressions **always** results in a rational expression.

Complex algebraic fractions have **rational** expressions in the numerator, the **denominator**, or both.

Example

Rewrite the fraction as a **rational** expression.	$\dfrac{\dfrac{2x+1}{x^2-25}}{\dfrac{4x^2-1}{x-5}} = \dfrac{2x+1}{x^2-25} \div \dfrac{4x^2-1}{x-5}$
To divide, **multiply** the first expression by the **reciprocal** of the second divisor.	$= \dfrac{2x+1}{x^2-25} \cdot \dfrac{x-5}{4x^2-1}$
Factor the **quadratic** expressions in the **denominators** of the rational expressions.	$= \dfrac{2x+1}{(x-5)(x+5)} \cdot \dfrac{x-5}{(2x-1)(2x+1)}$
Combine the factors to create a single **fraction**.	$= \dfrac{(2x+1)(x-5)}{(x-5)(x+5)(2x-1)(2x+1)}$
Cancel out the **common** factors to get the **simplest** form.	$= \dfrac{1}{(x+5)(2x+1)}$