Solving Systems of Linear Equations Algebraically

Objective

In this lesson, you will

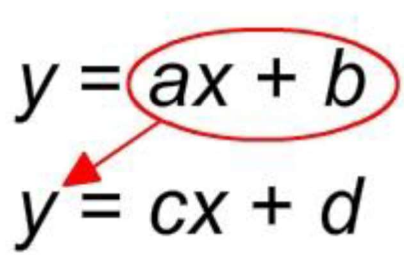
solve a system of linear equations algebraically.

Substitution

The solution to a system in two variables is a pair of *x*- and *y*-coordinates that satisfies both equations.

In both equations, y is equal to an expression.

Substitute the expression that is equal to *y* in the first equation into the second equation, or vice versa.



→ This will eliminate *y* from the resulting equation. Once *y* is gone, we can solve for x.

Lesson Activity The two equations that model the two membership options are: $c = 30m + 50$ and $c = 20m + 100$.

A. To find out when the cost will be equal, you will need to solve for *m* using substitution. Substitute the expression for the cost from the first equation into the cost, *c*, in the second equation.

$$c = 30m + 50$$
$$c = 20m + 100$$
$$30m + 50 = 20m + 100$$

B. Solve the equation in part A for *m*.

$$30m - 20m + 50 = 20m - 20m + 100$$
$$10m + 50 = 100$$
$$10m + 50 - 50 = 100 - 50$$
$$10m = 50$$
$$m = 5$$

C. Would you get the same solution if you substituted the expression for cost in the second option for *c* in the first equation? Why?

Yes, the solution is the same because the equation created using this method is the same as the equation in the first method. It just flips the two sides: 20m + 100 = 30m + 50.

The solution for a system of equations is a pair of values.

We can find its value by substituting the value of one variable into either of the two original equations.

Lesson Activity

Finding the Second Variable

A. You found that the value of *m* is 5 when the values of *c* are equal to each other. Substitute this value into the first equation and solve for *c*. What is the value of *c*?

$c = 30m + 50$

$c = 30(5) + 50$

$= 150 + 50$

$=$ **200**

The value of *c* is **200**.

B. Now substitute the value of *m* into the second equation and solve for *c*. What is the value of *c*?

$c = 20m + 100$

$c = 20(5) + 100$

$= 100 + 100$

$=$ **200**

The value of *c* is **200**.

C. What is the solution for the system of equations? What does the solution mean in terms of the situation?

The solution is *m* = **5** and *c* = **200**.

This means that after **5** months, the total cost of membership will be $**200** for both plans.

Example:

$x = \frac{2}{3}y - 3$ Equation (1) has *x* alone on one side.

$5y + 3x = 5$ Equation (2) ○ is ◉ is not in slope-intercept form.

→ The substitution method is still ideal for solving this system.

$5y + 3\left(\frac{2}{3}y - 3\right) = 5$ Substitute the expression equal to *x* from equation (**1**) into equation (**2**).

$5y + 2y - 9 = 5$ To solve the problem, isolate **y**.

$7y = 14$

$y = 2$

→ add **9** to both sides of the equation

→ divide both sides of the equation by **7**

$5(2) + 3x = 5$ To find the *x*-value, **substitute** the *y*-value into one of the original equations.

$3x = -5$

$x = -\frac{5}{3}$

The solution to the system of linear equations is: $x = -\frac{5}{3}$, $y =$ **2**.

Elimination

When neither equation has a variable alone on a side, the elimination method helps us solve the system.

To use the elimination method, a variable's coefficient in the two equations must be the **same** number but with **opposite** signs.

Example:

Modify one of the equations by ○ adding ◉ multiplying both sides by a constant.

$$5x + 2y = 9$$
$$x + 6y = 13$$

Choose the constant that makes the coefficient of one of the variables have the **same** number but with the **opposite** sign as the coefficient of the same variable in the other equation.

$$-3 \cdot (5x + 2y) = -3 \cdot 9$$
$$x + 6y = 13$$

○ Adding ◉ Multiplying equation (1) by -3 makes the coefficient of *y* equal to -6.

$$-15x - 6y = -27$$
$$+ \quad x + 6y = 13$$
$$-14x = -14$$
$$x = 1$$

After getting the modified equation, add it to the other original equation.

→ cancel out *y*, the variable with **opposite** coefficients in the two equations

→ Solve for *x*.

To find *y*, substitute the *x*-value in equation (2).

$$1 + 6y = 13$$
$$1 + 6y - 1 = 13 - 1$$
$$6y = 12$$
$$y = \mathbf{2}$$

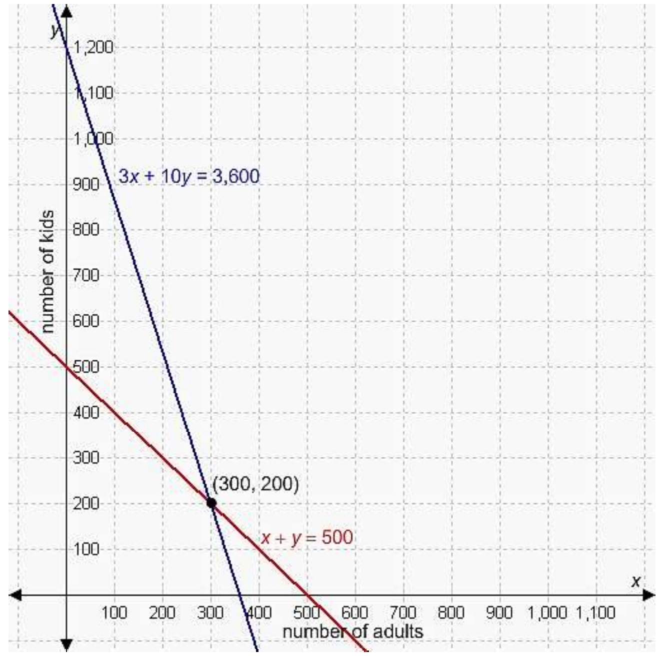
So, the solution to this system of linear equations is *x* = **1**, *y* = **2**.

Elimination is always performed using the same four steps:

1. Before performing elimination, make sure the equations have a variable with **opposite** coefficients.
2. ◉ Add ○ Subtract the modified equation to the other original equation.
3. Solve for the value of the variable that is not eliminated.
4. Use this variable's value to find the value of the variable **eliminated** earlier.

We've used two algebraic methods for solving a system of equations—substitution and elimination. Choose whichever method is easier for solving a particular system.

- When graphing a system of equations, the **intersection** point of the two lines represents the system's solution.
- Not all systems of equations have a **single** solution.
- Some systems have **no** solution, and some have an **infinite** number of solutions.



Example:

$$x - 3y = 8$$
$$x - 3y = 6$$

In the given system of equations, the **left side** of each equation contains the same expression.

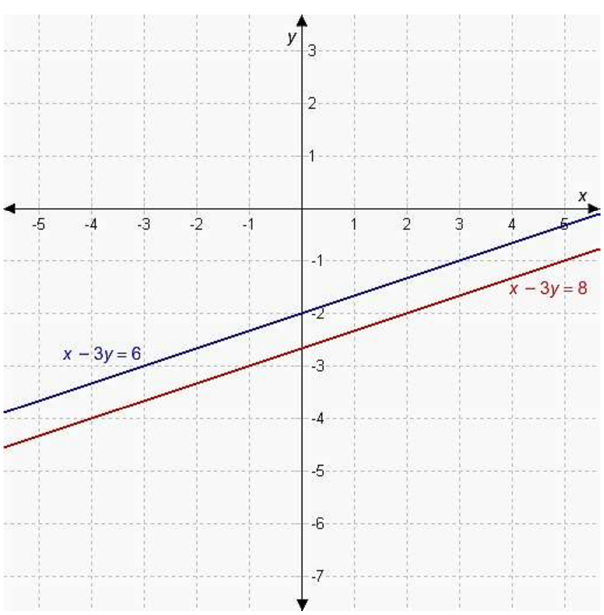
The same linear expression cannot equal both **8** and **6**.

→ This system has **no solution**.

$$\begin{aligned} x - 3y &= 8 \\ -x + 3y &= -6 \\ \hline 0 &= 2 \end{aligned}$$

Suppose we try to solve the system of equations by elimination. Adding the two equations would cancel out both the variables and give us an equation with different numbers on either side.

This equation ○ is ◉ is not true which shows that there is **no** solution to the given system.



Notice the two lines you get when graphing the two equations will never intersect.

The lines are **parallel**.

The graph confirms that this system has **no solution**.

Example:

$$x - 3y = 8$$
$$-2x + 6y = -16$$

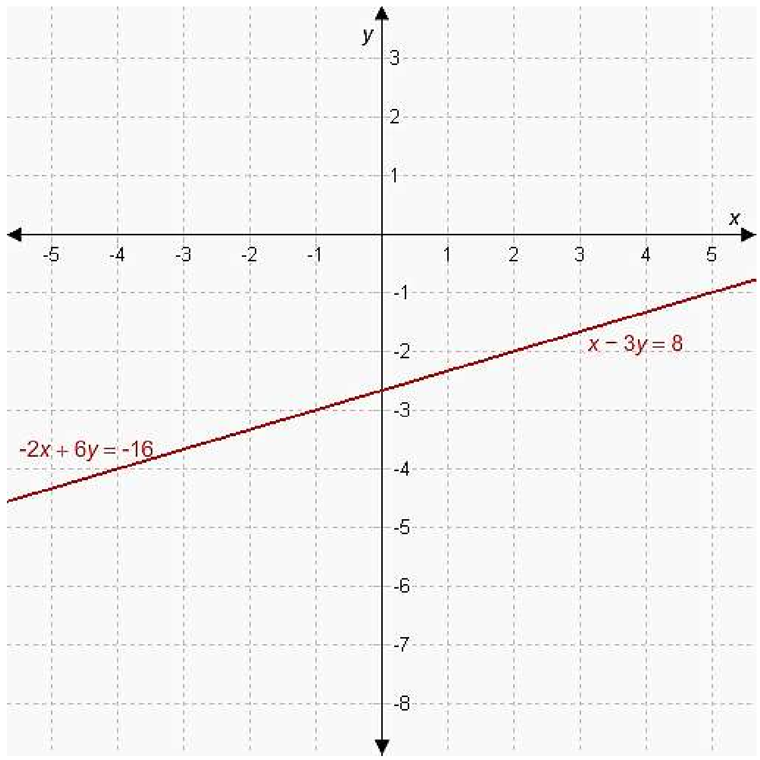
To solve this system using the elimination method, we would multiply the first equation by 2.

Adding the two equations would cancel out both variables and result in an equation with the same number on either side.

$$2x - 6y = 16$$
$$-2x + 6y = -16$$
$$0 = 0$$

The graph of this system of equations gives the same line for both equations.

The graph confirms that this system has infinite solutions.



Summary

The two methods discussed in this lesson for solving systems of linear equations algebraically are substitution and elimination. Does it matter which method is used to solve the system of equations? How would you determine which method to use?

answers will vary