Solving Linear Equations

Objective

In this lesson, you will

solve linear equations with rational coefficients.

Linear Equations

We can use linear equations to model real-world situations.

For more complex equations:

- need to repeatedly perform **opposite** operations on both sides of the equation
- may need to apply the Associative, **Commutative**, and Distributive Properties of operations to **isolate** variables and then solve for those variables.

Example:

Alec, Ben, and Cedric collect coins. Alec has 5 less than $1\frac{2}{3}$ times the number of coins Ben has.
Cedric has 12 more than $1\frac{1}{2}$ times the number of coins Ben has.
If Alec and Cedric have the same number of coins, how many coins does Ben have?

To solve this problem:

1. Assign a **variable** to the quantity that is common to the descriptions of the other quantities.
   - The number of coins **Ben** has is common to the descriptions of the coins the other boys have.
   - Let b represent the number of coins **Ben** has.
2. Form expressions for the other quantities that depend on the variable.
   - Alec's coins: $\frac{5}{3}b - 5$
   - Cedric's coins: $\frac{3}{2}b + 12$
3. Create an **equation** by using the relationship between the two expressions.
   - Alec and Cedric have the same number of coins. So, set the expressions equal to each other.

$$\frac{5}{3}b - 5 = \frac{3}{2}b + 12$$

4. Perform opposite operations and apply the properties of operations to get the terms with variables on one side and the numeric terms on the other side of the equation.

$$\frac{5}{3}b - \frac{3}{2}b = 12 + 5$$

5. Solve the resulting equation by using the Distributive Property and opposite operations where needed:

$$\frac{5}{3}b - \frac{3}{2}b = 12 + 5$$

$$\frac{1}{6}b = \underline{17}$$

$$b = \underline{102}$$

So, Ben has 102 coins.

We can plug the variable's value into the expressions to find the value of the other quantities.

→ Alec's coins: $\frac{5}{3}b - 5 = \frac{5}{3}(102) - 5 = 170 - 5 =$ 165

→ Cedric's coins: $\frac{3}{2}b + 5 = \frac{3}{2}(102) + 12 = 153 + 12 =$ 165

Alec and Cedric have the same number of coins.

It's a good idea to recheck the work at the end.
For example, if the number of coins had been different that would indicate a mistake had been made.

Combining Like Terms

Like Terms		
$2x$ and 3x	$\frac{1}{2}b$ and $\frac{2}{3}b$	$6.3d$ and $4d$
4 and 5	3.4 and 2	$\frac{1}{6}$ and $\frac{2}{5}$

In a single-variable equation, there are terms with a variable and terms without a variable.
These two sets of terms are like terms.

To solve a linear equation, we need to combine the like terms.

Example: Let's combine like terms in an expression.	$3x - 2 + 4x + 7$
The terms $3x$ and 4x are like terms; so are the terms -2 and 7.	$3x + (-2) + 4x + 7$
Apply the Commutative Property to get the like terms side by side.	$3x + 4x + (-2) + 7$
Use the Associative Property to group the terms.	$(3x + 4x) + (-2 + 7)$
Use the Distributive Property to combine the like terms with a variable.	$(3 + 4)x + (-2 + 7)$
	7 x + 5

Lesson Activity

Gavin's goal is to bike about 65 total miles over four days. Each day, he wants to ride 1.5 times as far as he rode the day before.

If x is the number of miles Gavin bikes on the first day:

- The expression for the distance he should bike on the second day is **1.5** x.
- The expression for the distance he should bike on the third day is (1.5)(**1.5** x) = **2.25** x.
- The expression for the distance he should bike on the fourth day is (1.5)(**2.25** x) = **3.375** x.

Since his goal and the total distance he should bike over the four days must be equal:

$65 = x +$ **1.5** $x +$ **2.25** $x +$ **3.375** x

which simplifies to $65 =$ **8.125** x

So, $x =$ **8** → Gavin should ride **8** miles the first day.

The table shows the distances Gavin should ride each day and his goal.

Day	Distance Biked
1	*8 miles*
2	**12 miles**
3	**18 miles**
4	**27 miles**
Goal:	*65 miles*

One way to check the answer and make sure the calculations were done correctly is **adding** up the miles Gavin rode each day to determine whether they **totaled** 65 miles.

Question

Solve the equation for y. $6.6y + y + 6.5 = y - 6.7$

1. Subtract **y** from both sides. $6.6y + 6.5 =$ **-6.7**
2. Subtract **6.5** from both sides. $6.6y =$ **-13.2**
3. Solve for y. $y =$ **-2**

Summary

How does the Distributive Property help when combining like terms to solve the equation $4x + 6x = 30$?

answers will vary